Theoretical Base for the Kenya Macro Model: The KIPPRA-Treasury Macro Model

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Abstract

This paper provides a birds-eye view of the theoretical underpinnings of the KIPPRA-Treasury macroeconomic model. The model is built mostly along the now fairly standard lines of the aggregate demand–aggregate supply framework. The model is demand driven in the short run, with multiplier effects through consumption and investment and the external sector. An important assumption of this model is that any demand is actually met; that is, we assume that the price system ensures that there is always some excess capacity in the economy. This is justified by the liberalized nature of the Kenyan economy. The model is designed in such a way that it has a tendency to return to equilibrium with 'normal' capacity utilization and unemployment rates in the medium and long run. The main feedback mechanisms in the real economy work through the wage–price spiral, the interest rate and the real exchange rate. The paper is also accompanied by an extremely simplified and elaborated annex that contains step-by-step derivation of major equations of the model. This is aimed at making the model accessible to a wide audience.

Acknowledgements

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Contents

Abbreviations

CES constant elasticity of substitution

GDP gross domestic product

PPP purchasing power parity

1 Introduction

As part of the ongoing reforms in economic management in most developing countries, including Kenya, it is expected that policy-making and budget preparation in them will be based on the Poverty Reduction Strategy Paper (PRSP). This in turn will be closely tied with the Medium-Term Expenditure Framework (MTEF). Both the realization of PRSP and the use of MTEF require an overall macroeconomic framework that ensures consistency in defining the aggregate resource envelope and forecasting major macro aggregates three to four years ahead. The macro model is an invaluable instrument for achieving that. Since both preparing the budget and forecasting key macro variables are done in a consistent manner, alteration of the components of the budget in a discretionary manner is not an option (that is, without taking the overall consistency framework into account).

Another important justification for having a macro model is its capability to simulate the effect of policy options. This is crucial for policy-makers because it helps them assess the implications of a proposed policy or packages of policies before they are implemented. Policy analysis conducted with the aid of such models avoids a partial analysis, and hence partial understanding, of issues of national significance. The analysis takes into account all possible links in the economy that are not easily traced with other approaches.

Macro models are also used to carry out macroeconomic research. Macroeconomists at research institutions can use such a model to investigate a wide range of issues such as external shocks and domestic responses and the implications of alternative policies. In the process, they should gain a better understanding of the modelled economy. This in turn improves the model and hence policy formulation.

2 An Overview of the KIPPRA-Treasury Macro Model Theory

2.1 The real economy

Figure 1 provides an overview of the real side of the model. The dark grey boxes denote agents, the light grey boxes, markets, and the arrows denote transactions.

The model contains four types of agents and three markets:

Agents	*Markets*
1) domestic production[1]	1) labour market
2) households,	2) product market
~~2)~~3) government	3) financial market
~~2)~~4) rest of the world	

The arrows in figure 1 indicate the transactions between the agents. The direction of the arrows gives the direction of payment. For example, there are two arrows drawn between the households and the government: households pay direct taxes to the government and the government provides transfer payments to households.

We will now briefly discuss the economics in figure 1, starting with the 'domestic production sector' on the left side of the figure. By definition, all productive activity takes place in this sector. It includes private firms, parastatals and the public service sector. The domestic production sector may be subdivided into major sectors of the economy, depending on size and data availability. Output (value added) is produced according to a constant elasticity of substitution (CES) production function with capital and labour as inputs. This production

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[1] Private firms, parastatals and public service sector.

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Figure 1. Overview of the real model (arrows indicate direction of payments).

function may differ by sector. Production leads to a demand for labour and a demand for investment goods. Labour demand and investment are, therefore, related foremost to output. In addition, they are related to relative factor costs, that is, the relative prices of capital and labour, and to profitability. The demand for labour is directed to the labour market, the demand for investment goods to the product market. In addition to labour and capital costs, the production sector pays indirect taxes and corporate taxes to the government and receives subsidies from it. Part of the remaining profits is distributed to households as corporate or non-wage income.

In the labour market, the demand for labour is confronted with the supply of labour. Labour supply is determined by demographic factors, education, the unemployment rate (proxying the discouraged-worker effect) and the net real wage. The first two factors are exogenous in the model and the latter two endogenous. The wage rate is determined by a bargaining model, in which prices and the unemployment rate play a major role. The working of the informal labour market is still rudimentary, as there is little understanding of how it functions.

Household income consists of wages and corporate or non-wage income, such as self-employment income, plus government transfers. After the household pays direct taxes, disposable income remains, the main determinant of consumption demand. Wealth and the interest rate may also play a role. Disposable income minus consumption equals household savings.

The government receives direct taxes from households, corporate and indirect taxes (net of subsidies) from the production sector, and aid from the rest of the world. Aid is defined as grants plus the grant component of concessional loans.[2] Government spending consists of transfers to households and government expenditure. Tax rates are exogenous, but the tax base and thus tax income is endogenous. Aid from the rest of the world is exogenous. Transfers to households and government expenditure may be related to GDP or exogenous, that is, policy determined. Exceptions are payments that depend on the state of the economy and statutory obligations.

[2] In practice, grants are not included in the calculation of the budget deficit.

The government deficit follows as the difference between tax and grant income and total government spending.

Exports to the rest of the world are determined by world trade, the real exchange rate and investment-to-GDP ratio. Profitability, which is particularly important for the sectors in which products are traded according to world prices, such as coffee and tea, is implicitly assumed to be proxied by the investment–GDP ratio.

Total demand equals the sum of investment, consumption, government expenditure and exports. This demand enters the product or goods and services market. Supply of goods and services comes from the rest of the world (imports) and from the production of the domestic production sector (value added). The price of imports is an exogenous world price. The demand for imports is modelled as a function of total demand and the real exchange rate. Total demand minus imports equals GDP at market prices, which is produced by the domestic production sector. The price of production depends on production costs including the cost of intermediate imports and the capacity utilization rate.

For each agent the difference between income (the incoming arrows) and spending (the outgoing arrows) equals savings. These savings may be negative, in which case they are deficits. This will generally be the case for the government. The savings flow into the financial markets, the light grey box at the bottom of the figure. So the savings of the domestic production sector, the households, the government and the rest of the world all get together in the financial market, where the appropriate bonds and loans are exchanged. By definition, these four savings add up to zero, so the domestic savings (domestic production sector, government plus households) are equal to the negative of the foreign savings. Note that if the government savings are negative, that is, if the government runs a deficit, funds (loans) will flow from the financial market to the government to finance the deficit.

2.2 The nominal model

Figure 2 gives an overview of the nominal economy. The light grey boxes denote demand and supply in the labour, goods and services, and the money market. The medium grey boxes denote endogenous prices and the dark grey ones exogenous variables. The arrows indicate direction of causation or determination.

Six prices are determined endogenously in the model:

- price of goods and services
- nominal wage
- real wage
- nominal exchange rate (the price of foreign exchange)
- real exchange rate
- domestic nominal interest rate (the 'holding' price of money)

Wages and prices are determined in the labour and product markets, as indicated in the previous section. Wages and prices also depend on each other as indicated by the double arrow, so there is a wage–price spiral in the model. The exchange rate and the interest rate are determined in the financial market. The financial market is subdivided into the markets for domestic money and domestic bonds and the market for foreign assets. By Walras's law we have to model only two of these markets, and if these are in equilibrium, so is the third. We model the markets for domestic money and for foreign assets and leave

[full page]

Figure 2. The nominal model (arrows denote causation).

the market for domestic bonds implicit. We assume that the exchange rate is floating, so the money supply is available as an exogenous policy instrument. Aggregate demand, the price level and the interest rate determine money demand. The interest rate moves to clear the money market, so the interest rate is a function of money supply, real demand and prices.

The exchange rate clears the market for foreign assets. A rise in the domestic interest rate relative to foreign interest rates makes domestic assets relatively more attractive and thus causes an appreciation. The real exchange rate follows by definition from the nominal exchange rate, the foreign price level and the domestic price level.

2.3 The main feedback mechanisms

The model is built mostly along the now fairly standard lines of the aggregate demand–aggregate supply framework. The model is demand driven in the short run, with multiplier effects through consumption and investment. We assume that any demand is actually met; that is, we assume that the price system ensures that there is always some excess capacity in the economy. This is a good reflection of the Kenyan economy since the mid-1980s in general and after 1993 in particular. High demand leads to high-capacity utilization rates of capital and low unemployment rates, however, which lead to wage and price increases. Assuming that the resulting inflation is not accommodated by an increase in the growth rate of money, the higher inflation leads to higher interest rates and a real appreciation, causing a reduction in investment and exports. In this way, the model has a tendency to return to equilibrium with 'normal' capacity utilization and unemployment rates in the medium and long run.

These main feedback mechanisms in the real economy working through the wage–price spiral, the interest rate and the real exchange rate are also illustrated in figure 2. For instance, an increase in aggregate demand raises labour demand, reduces the unemployment rate, raises wages and starts a wage–price spiral. The resulting inflation causes real appreciation, reduction in competitiveness and reduction in exports. In addition, the demand for money increases, with rising interest rates and lower investment as a result.

The drop in exports and investment reduces demand again, until equilibrium is restored.

Even though total demand may be stabilized in this way, the feedback mechanism may well change the composition of demand. For instance, if the original increase in demand came from an increase in government spending, the net result will be a shift from exports and investment to government spending, resulting in a government deficit and a current account deficit.

Moreover, this mechanism works only if the money supply is not accommodating. If the money supply is raised in response to inflation, and raised just enough to meet the increase in the demand for money, domestic interest rates will not rise, and the reduction in demand caused by higher interest rates is also neutralized. If money supply is raised even further, the nominal exchange rate will depreciate and part or all of the earlier real appreciation will be neutralized as well. Now the increase in demand is no longer countered by the feedback mechanisms, and inflationary pressure will continue. So, a policy of targeting the exchange rate so as to maintain purchasing power parity (PPP) is highly cyclical, causing large swings in the economy. As noted in the monetary section below, Kenyan government's policy seems to target exchange rate and inflation stability using the Treasury bill rate as an intermediate instrument.

3 The Theory behind the Behavioural Equations

3.1 Price determination

Price determination for a single good

We assume that output prices are set by firms who operate in a market structure of monopolistic competition. That is, we assume that for each good there exists an inverse demand curve $p^f = p^f(z)$, with p^f denoting the price at factor cost and z the gross output. The price at factor cost is exclusive of indirect taxes and subsidies, and thus it equals the price the firm actually receives for its product. We also assume that there exists a well-behaved cost function $c = c(z)$. Profit maximization then leads to

$$argmax(z): (1 - t_{\pi})\left[p^f(z)z - c(z)\right] \qquad [1]$$

where t_B is the profit tax. Profits are maximized by setting the price p^f equal to

$$p^f = \left(1 - \frac{1}{\varepsilon}\right)^{-1} mc \qquad [2]$$

where ε is the price elasticity of demand and *mc* denotes marginal cost: $mc = dc/dz$. Note that the profit tax has no influence on the price, since both marginal revenue and marginal cost are reduced by the same amount. The market price, denoted p_y, is related to the factor cost price p^f by

$$p_y = p^f(1 + t_z - s_z) \qquad [3]$$

where t_z and s_z are the indirect tax and subsidy rates. The relation for the market price is therefore

$$p_y = \left(1 - \frac{1}{\varepsilon}\right)^{-1}(1 + t_z - s_z)\, mc\,. \qquad [4]$$

We do not observe marginal cost, however, so the above equation is not operational. Therefore, we first define capacity as the level of production with minimum average long-run cost. Note that marginal cost at capacity equals this average long-run cost. Using duality theory, we make a Taylor expansion of marginal cost around capacity output. We get

$$mc = ac + \alpha z^* \frac{(z - z^*)}{z^*} \qquad [5]$$

where z^* equals capacity level of output and α involves derivatives of the cost function. The term z/z^* is the capacity utilization rate, denoted q.

Average cost, denoted ac, equals

$$ac = \frac{w(1 + s_f)l + p_k K + p_m m}{z} \qquad [6]$$

where w denotes the gross wage level, s_f the social security contributions rate paid by the firm, l employment, p_k the user cost of capital, K the capital stock, p_m the domestic currency price of imports and m imports. The user cost of capital p_k in turn equals

$$p_k = (i - \pi + \delta + \bar{r})\ p_i$$

[7]

$$= (r + \delta + \bar{r})\ p_i$$

where i is the nominal interest rate, π the inflation rate, δ the depreciation rate, r the real interest rate and $\bar{r}$ the risk premium; p_i is the price of investment goods, that is, the price of capital when bought.

The capacity utilization rate q should ideally be constructed using a proper explicit production or cost function. For the moment, we simply assume that output fluctuates around capacity, so that we may approximate capacity by average output. This implies

$$\frac{z - z^*}{z^*} \ = \ q - 1 \ = \ \frac{z}{z_{average}} - 1. \qquad [8]$$

The basic equation for the percentage change in value-added price is then

$$\hat{p}_y \ = \ \hat{a}c \ + \ \beta_1 \ \Delta(q - 1) \ + \ \beta_2 (q - 1) \ + \ \frac{\Delta t_z - \Delta s_z}{1 + t_z - s_z} + constant \qquad [9]$$

The equation contains a constant. This is true for all behavioural equations, but for brevity's sake we leave the constants out from now on. Assuming technological progress to be labour saving, and denoting labour productivity by h, we get

$$\hat{a}c \ = \ \alpha_l \ (\hat{w} - \hat{h}) \ + \ \alpha_k \ \hat{p}_k \ + \ \alpha_m \ \hat{p}_m \qquad [10]$$

where α_j is the share of factor j in gross output.

Influence of competitor prices

So far we have abstracted from the influence of competitor prices. In the long run, competitor prices should not matter much, because in equilibrium all firms will have a certain competitive rate of return on their investment. So if, say, exporting firms would continually meet foreign price reductions without accompanying reductions in their own costs, they would end up going bankrupt and disappear. In the short run, however, firms may well follow competitor prices in order to retain their market shares if foreign prices fall, or if foreign prices rise, to get a short-run increase in profits when expansion of output is not possible in the short run. There is, indeed, strong evidence that export prices especially are sensitive to foreign competitor prices.

We may model this by writing

$$\hat{p}_y^{\,m,s} = (1-\gamma)\,\hat{p}_y + \gamma\,\hat{p}_{comp}$$
$$= \hat{p}_y + \gamma(\hat{p}_{comp} - \hat{p}) \qquad [11]$$

where the superscript *m,s* indicates short-run market prices and γ is the elasticity of final demand prices to competitor prices. Note that if $\gamma \neq 0$, the value-added price, p_y, also depends on competitor prices; a reduction in price in order to meet a reduction in competitor prices leads to a reduction in value added and profits.

All together, we get for the final goods prices determined in the market sector:

$$\hat{p}_y^{\,m,s} = (1-\gamma)\begin{pmatrix} \alpha_w(\hat{w}-\hat{h}) + \alpha_k\,\hat{p}_k + \alpha_m\,\hat{p}_m + \beta_1\Delta q \\ + \beta_2(q-1) + \dfrac{\Delta t_z - \Delta s_z}{1+t_z-s_z} \end{pmatrix} + \gamma\; p_{comp} \qquad [12]$$

3.2 Price of aggregate goods

Aggregate goods basically follow the same structure as given above for single goods. We only have to take into account that some goods are set not according to market conditions but directly by the government. Let the share of such goods be λ. Thus, we get for the price equations of aggregate goods, $\hat{p}^{\,a}$:

$$\hat{p}^a = (1-\lambda)\,\hat{p}_y^{m,s} + \lambda\,\hat{p}^p \qquad [13]$$

where the superscript *p* denotes policy determined.

Three aggregate goods prices are in the model: one for consumer goods, one for investment goods, and one for export goods. The straightforward thing to do is to model all three according to the above equations with coefficients that are appropriate for the separate aggregate goods. However, we do not know much about the user cost of capital p_k. Therefore, we substitute it out of the price equations. To do this, we assume that the depreciation rate is constant, which also shows the practice of the Government of Kenya, so that

$$\hat{p}_k = \frac{d(r+\delta+\bar{r})}{r+\delta+\bar{r}} + \hat{p}_i = \frac{dr}{r+\delta+\bar{r}} + \hat{p}_i . \qquad [14]$$

On the further assumption that effective indirect taxes and subsidies are zero (for instance, because indirect taxes paid on investment goods may be taken as a credit by the investing firm) and also setting γ equal to zero for investment goods, we get

$$\hat{p}_i = \alpha_{w,i}(\hat{w} - \hat{h}) + \alpha_{k,i}\,\hat{p}_k + \alpha_{m,i}\,\hat{p}_m + \beta 1_i\,\Delta q + \beta 2_i\,(q - 1) \qquad [15]$$

where the subscript *i* stands for investment goods. This implies

$$\begin{aligned}\hat{p}_k &= \frac{dr}{r+\delta+\bar{r}} + \alpha_{w,i}(\hat{w} - \hat{h}) + \alpha_{k,i}\,\hat{p}_k + \alpha_{m,i}\,\hat{p}_m \\ &\qquad + \beta 1_i\,\Delta q + \beta 2_i\,(q - 1) \\ &= \frac{1}{1 - \alpha_{k,i}}\left[\begin{array}{l}\dfrac{dr}{r+\delta+\bar{r}} + \alpha_{w,i}(\hat{w} - \hat{h}) + \alpha_{m,i}\,\hat{p}_m + \\ \beta 1_i\,\Delta q + \beta 2_i\,(q - 1)\end{array}\right]\end{aligned} \qquad [16]$$

We may use this equation to substitute out the user cost of capital in the price equations. Substituting this equation into the equation for the change in short-run market prices of good *j*, $\hat{p}_j^{m,s}$, we get

$$\hat{p}_j^{m,s} = (1-\gamma_j)\left[\begin{array}{l} \alpha'_{r,j}\dfrac{dr}{r+\delta+\bar{r}} + \alpha'_{w,j}(\hat{w}-\hat{h}) \\ + \alpha'_{m,j}\,\hat{p}_m + \beta 1'_j \Delta q + \beta 2'_j (q-1) \\ + \dfrac{\Delta t_{z,j} - \Delta s_{z,j}}{1+t_{z,j}-s_{z,j}} + \gamma_j\ \hat{p}_{comp} \end{array}\right] \quad [17]$$

where $\alpha' r, j = \alpha_{k,j}\dfrac{1}{1-\alpha_{k,i}}$ $\qquad \alpha'_{w,j} = \alpha_{w,j} + \alpha_{k,j}\dfrac{\alpha_{w,i}}{1-\alpha_{k,i}}$

$$\alpha'_{m,j} = \alpha_{m,j} + \alpha_{k,j}\frac{\alpha_{m,i}}{1-\alpha_{k,i}}$$

$$\beta 1'_{j'} = \beta 1_j + \alpha_{k,j}\frac{\beta 1_j}{1-\alpha_{k,i}} \qquad \beta 2'_j = \beta 2_j + \alpha_{k,j}\frac{\beta 2_j}{1-\alpha_{k,i}}$$

In the above equation we assumed that the capacity utilization rate q is modelled at the macro level; $\alpha'_{w,j}$ has the interpretation of the cumulated labour share in good *j,* including the labour content of the capital stock. Similarly, $\alpha'_{m,j}$ has the interpretation of the cumulated share of imports in the production of good *j*. The above equation thus models production as ultimately using only labour, imports and time (the effect of *r*).

3.3 Wage determination

In any macroeconometric model the wage equation is of crucial importance. As noted in Karingi and Ndung'u (2000) this equation should be able to capture the effects of unemployment if it follows the Phillips curve approach, or the effects of taxes, productivity, real exchange rate, and so on, if it follows the Layard-Nickell approach (Layard and Nickell 1985). In fact, with the liberalization of wage guidelines in Kenya allowing workers and employers more freedom in wage negotiations, a bargaining approach to wage determination following the work of Layard and Nickell (1985) would be appropriate. It needs to be mentioned at this point that wage formation in Kenya may be at least at three levels (unionized, competitive and administered). The observed wage would be a function of the competitive, administered and bargained. Thus, our model may describe only a segment of the labour force in the formal sector. To be sure, it is important to check whether bargaining really takes place or employers have absolute power. In

1999, 328 collective agreements were registered by the industrial court, representing 113,758 unionizable workers (Kenya Central Bureau of Statistics 2001). In the year 2000, the number of agreements declined to 316 representing 71,586 unionizable employees. It is therefore clear that there are formal collective bargaining agreements as assumed in this model framework, and most employees negotiate wages with their employers. Since bargaining takes place every time, a bargaining framework is not out of place. Assuming that bargaining does indeed take place and assuming further that a successful increase owing to bargaining is also reflected in administered wage sectors, as employers avoid worker turnover and try to prevent unionization, a Nash bargaining solution can be found. The theory therefore hinges on the assumption that benefits from bargaining benefit not only those in unions but spill over into the rest of the formal sector.

Therefore, wage determination in the KIPPRA-Treasury Macro Model is through a model of bargaining. As indicated above, the bargaining can take place at the level of centralized or decentralized unions or at the level of individual workers. The common idea is that workers and firms have a joint surplus and they have to come to some agreement as to how to divide this surplus. The surplus is joint, because both the firm and its workers have some market power over it, as a result of their specific skills, the legal set-up, or other reasons.

The starting point of the bargaining model consists of defining what the different parties care about. Negotiations are generally about gross wages, but it is important to realize that neither the workers nor the firms care about gross wages directly.

To illustrate this and to set up the bargaining model, we assume a simple constant returns-to-scale production function with labour only: $y = hl$, where y equals production (value added), h productivity and l labour input. Profits are

$$\Pi = p_y y - w(1+s_f)l \qquad [18]$$

where Π denotes profits, p_y the output (= value added) price, w the wage rate and s_f the taxes, social security contributions, pension benefits, and so on, paid by the firm that are associated with labour. Since the production function is constant returns to scale, we may divide by labour and the value-added price and thus calculate the real profit rate per worker:

$$\frac{\Pi}{p_y\ l} = \frac{y}{l} - \frac{w(1+s_f)}{p_y} = h - \frac{w(1+s_f)}{p_y} \qquad [19]$$

The last term in the above equation is called the real product wage, denoted w_y

$$w_y = \frac{w(1+s_f)}{p_y} \qquad [20]$$

This is the wage concept the firm cares about. If it is below *h* (productivity), the firm makes a profit; otherwise a loss.

The workers care about the purchasing power of the wage in terms of consumer goods. This consumption wage, denoted w_c, is defined as

$$w_c = \frac{w(1-s_l)}{p_c} \qquad [21]$$

where s_l denotes the direct taxes and social security contributions paid by labour and p_c the consumer price.

If we divide the wage costs to the firm by the wage benefit to the worker, we get the so-called wedge, denoted Λ. The formula is

$$\Lambda = \frac{w_y}{w_c} = \frac{(1+s_f)}{(1-s_l)} \frac{p_c}{p_y} \qquad [22]$$

We can go a little further and note that $p_c = p_d(1 + t_z)$, where t_z is the indirect sales tax (net of subsidies) and p_d is the domestic goods price to the firm. The latter is a weighted average of the value-added price and the price of imports: $p_d = p_y^{1-a} p_m^{(a)}$, where a is the share of imports in output.[3] This implies that

[3] Here, in principle, linear weights can be used, although they tend to swing prices to extremes. Non-linear weights such as this will dampen this tendency to pull the average away.

$$\frac{p_c}{p_y} = \left(\frac{p_m}{p_y}\right)^{\alpha} (1+t_z)\,. \qquad [23]$$

This means that the wedge has four components to it: taxes and social security contributions paid by the worker, social security contributions paid by the firm, indirect taxes, and the ratio of import prices to value-added prices. From points of view of workers and of firms, they are all the same, namely a 'tax' on labour. It is therefore to be expected that, at least in the long run, they all have the same effect in the negotiations.

This can be shown formally by considering the Nash bargaining solution to the wage negotiation process. This is the solution to

$$argmax(w):\ \left(\frac{p_y hl\ -\ w(1+s_f)l}{p_c}\right)^{\alpha} \left(\frac{w(1-s_l)}{p_c}\ -\ F\right)^{1-\alpha} \qquad [24]$$

where F is the fallback position to the workers in the negotiations. The solution is

$$w_y = (1-\alpha)h + \alpha F\Lambda\,. \qquad [25]$$

So, the wage is determined by three elements: productivity, the fallback position of workers, and the total wedge. Note in particular that the wedge enters as one single variable: all of its elements have the same coefficient.

The fallback position is generally considered proportionally to the average wage level and is also influenced by the open unemployment rate in the modern urban market. An important question that arises is whether the unemployment rate is the ideal fallback position. Given the large informal sector in the country, the informal sector wage may be a better one for workers. But given the scarcity of data regarding the sector, it may be difficult to obtain an informal sector wage series at the empirical stage.

The wages are proportional to h, and the log-linear first-order condition then becomes

$$\log w(1+s_f) = \log p_y + \log h + \beta_1 \log\Lambda - \beta_2 ur \qquad [26]$$

In terms of gross wages *w* we get[4]

$$\hat{w} = \beta_1 \hat{p}_c + (1 - \beta_1) \hat{p}_y + \hat{h} + \beta_1 \frac{\Delta s_l}{1 - s_{l_{-1}}} + (\beta_1 - 1) \frac{\Delta s_f}{1 + s_{f_{-1}}}$$

$$- \beta_2 \Delta ur - \beta_3 ur \qquad [27]$$

where the level of the unemployment is added partly because unemployment enters the theoretical equation in a highly non-linear way and in order to allow for a strong Phillips curve effect. In addition, the above equation captures the postulates of the Layard-Nickell model, as taxes and social security payments also play a part in the wage determination process.

3.4 Demand for factor inputs

We assume the generation of value added can be specified with a CES production function with capital and labour of the form:

$$y = \left(\alpha^{1+\rho} (l)^{-\rho} + (1 - \alpha)^{1+\rho} k^{-\rho}\right)^{-\frac{1}{\rho}}. \qquad [28]$$

Employment

The wage employment in the KIPPRA-Treasury model is determined from the CES production function postulated above. Different categories of employment need to be defined at the operational level of the model: public and private sector formal employment, self-employment, subsistence employment, and various types of informal sector employment. Modern-sector wage employment is modelled as a demand for labour. In previous Kenyan models such as the Chakrabarti model (see Kenya 1994; Alemayehu et al. 2001) the latter is taken as a function of economic activity (represented by real GDP) and the price of labour (real wage earning in the modern sector).

Non-military employment, simply called employment, is divided between the employees and the employers plus self-employed. The number of employers

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[4] Note that $dlog(1 + sf) \rightarrow dsf/(1 + sf-1)$, and that $\mathrm{dlog}(1-s_{l-1}) \rightarrow -ds_l/(1 - s_{l-1})$.

plus self-employed is exogenous in the model. The number of employees is derived from the CES production function above. The optimal labour input according to this CES production function is

$$l^{employees} = \left(\alpha^{1+\rho}\right)^{\delta} (Y^e) \left(\frac{w}{p_y}\right)^{-\sigma} \qquad [29]$$

where Y^e is the output produced with employees. Assuming Y^e to be proportional to *y,* we have in term of percentage changes

$$\hat{l}^{employees} = \hat{y} - \sigma(\hat{w} - \hat{p}_y) \qquad [30]$$

Total employment equals the sum of the number of employees and the number of employers plus self-employed l^{se}:

$$l = l^{employees} + l^{se}, \qquad [31]$$

so

$$\hat{l} = \alpha_e \hat{l}^{employees} + \alpha_f \hat{l}^{se} \qquad [32]$$

where α_e and α_f are the shares of the employees and employers plus self-employed in total employment.

Investment

The other factor input, capital stock, is modelled at the macro level. Different efforts have been made to model investment in Kenya. In some cases, investment has been divided into fixed and inventory (see Kenya 1994; Alemayehu et al. 2001). The fixed component is further divided across institutions (private, government, parastatal and traditional, the first component constituting more than 50% of total fixed investment in Kenya) and estimated for each of these institutions. Private investment was assumed to be determined by expectation of profit with the various proxies for profit having included GDP growth, export earning and real exchange rate. Moreover, the level of import (or the level of foreign exchange reserves), yields on government bonds (as cost of finance) and availability of credit are also taken as explanatory variables.

In other modelling attempts for Kenya, real investment spending is estimated for each production sector specified in the model (traditional, agriculture, manufacturing, services, government). Summing up such sectoral values gives total investment. The specification is fairly standard across sectors, and it follows a simple accelerator type model. The main variables used as explanatory variables are real GDP and real sectoral capital stock (see Keyfitz 1994; Alemayehu et al. 2001).

Private investment in the KIPPRA-Treasury model is specified in the context of a CES production function that has labour and capital as its arguments. The optimal macro capital stock is equivalent to the first-order solution to the production cost minimization subject to the CES technology:

$$k = (1-\alpha)y \left(\frac{p_k}{p_y}\right)^{-\sigma}. \quad [33]$$

From this we get

$$\frac{\Delta k}{k_{-1}} = \hat{y} - \sigma(\hat{p}_k - \hat{p}_y) \quad [34]$$

in percentage change form or

$$\frac{i}{k_{-1}} = \hat{y} - \sigma(\hat{p}_k - \hat{p}_y) + \delta. \quad [35]$$

Some authors suggest that the profit rate is also important. This may be justified by arguing that profits allow internal financing of investment, which is cheaper than external financing. This is a fairly common practice, especially among small and medium-size firms in Kenya. This is the main argument underlying the internal funds theory of investment. In addition, the capacity utilization rate may play a role as a direct indicator of the difference between optimal and actual capacity. Adding these elements, we get

$$\frac{i}{k_{-1}} = \hat{y} - \sigma(\hat{p}_k - \hat{p}_y) + \delta + \lambda\left(\frac{\pi}{k}\right)_{t-1} + \mu(q-1)$$

[36]

$$= \hat{y} - \sigma(\hat{p}_i - \hat{p}_y) - \sigma\frac{dr}{r + \delta + \bar{r}} + \delta + \lambda\left(\frac{\pi}{k}\right)_{t-1} + \mu(q-1)$$

where we use the fact that $p_k = (r + d + \bar{r})p_i$. Capital is often considered a quasi-fixed factor of production, because changing the capital stock takes a lot of time and involves important adjustment cost. Therefore, lags may be important in this equation. For the same reason, expectations matter as well, although they are difficult to model, and empirical models of investment with various expectations terms have had little success so far.[5]

Demand for imports

Kenya has a very open economy; as a result, the properties of the trade equations are key elements in determining the nature of any constraint in the balance of payments in a macroeconometric model. One such equation is the demand for imports. Gross output in the KIPPRA-Treasury model is formulated as a CES function of value added and imports. So we get a scale variable and price elasticity. For our scale variable we use the growth of gross output weighted by importance to total imports. We derive this as follows:

$$\begin{aligned} m &= m_c + m_i + m_g + m_x \\ &= \frac{m_c}{c}c + \frac{m_i}{i}i + \frac{m_g}{g}g + \frac{m_x}{x}x \end{aligned} \qquad [37]$$

where m_j denotes the cumulated imports content in final demand category j. Assuming the shares m_j / j to be constant, we get

$$\Delta m = \left(\frac{m_c}{c}\right)_{-1} \Delta c + \left(\frac{m_i}{i}\right)_{-1} \Delta i + \left(\frac{m_g}{g}\right)_{-1} \Delta g + \left(\frac{m_x}{x}\right)_{-1} \Delta x \text{ or} \qquad [38]$$

$$\hat{m} = \hat{m}_z \equiv \left(\frac{m_c}{m}\right)_{-1} \hat{c} + \left(\frac{m_i}{m}\right)_{-1} \hat{i} + \left(\frac{m_g}{m}\right)_{-1} \hat{g} + \left(\frac{m_x}{m}\right)_{-1} \hat{x}. \qquad [39]$$

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[5] For investment in inventories (*inv*) we could assume that firms want a constant inventory/sales (*z*) ratio that could be given by $\hat{inv} = \hat{z}$ or $\Delta inv = \left(\frac{inv}{z}\right)_{-1} \Delta z$.

This is the percentage change in imports caused by output effects, assuming constant import shares, that is, constant relative prices. To add the effect of relative prices we write

$$\hat{m} = \hat{m}_z - \sigma(\hat{p}_m - \hat{p}_y) \qquad [40]$$

where p_y denotes the value-added price.

Note that by implication, m_z is proportional to a geometric average of the components of gross output z:

$$m_z \propto c\frac{m_c}{m} + i\frac{m_i}{m} + g\frac{m_g}{m} + x\frac{m_x}{m}. \qquad [41]$$

It is worthwhile to add that the scale variable can also be obtained from the cumulative production structure derived from an input–output table of the Kenyan economy. However, this requires that a recent input–output table be available. If that is not the case, there is a need to find out whether the economy's structure has changed significantly from the most recent year when the input–output table was constructed. One can view the derived-scale variable of the cumulative productive structure as a check to the one derived from the geometric average of the components of gross output, and this is the approach adopted for the KIPPRA-Treasury Macro Model.

In empirical studies, it is often found that the elasticity of imports with respect to the scale variable m_z is larger than 1. This may be explained by a trend towards internationalization. We capture this effect with an additional parameter, $\alpha \geq 1$:

$$\hat{m} = \alpha\hat{m}_z - \sigma(\hat{p}_m - \hat{p}_y). \qquad [42]$$

The price of imports equals the (exogenous) price of imports in foreign currency, p_m(\$), times the exchange rate times 1 plus the import tariff rate:

$$\hat{p}_m = \hat{p}_m(\$) + \hat{e} + \frac{\Delta t_m}{1 + t_{m-1}} \qquad [43]$$

3.5 Labour supply and unemployment

Labour supply is modelled exogenously as the product of the population within working age times the labour activity ratio plus employment in non-working age. The equation for labour supply is

$$l^s = \alpha \; population_{working\ age} + employment_{non\ working\ age} \qquad [44]$$

where α is the exogenous labour participation ratio.

The labour supply minus the military service equals the economically active population l^{econ}:

$$l^{econ} = l^s - l^{military} \qquad [45]$$

where l^{econ} is the economically active population. The number of unemployed workers ***u*** is given as the economically active population minus employment:

$$u = l^{econ} - l\,. \qquad [46]$$

The unemployment rate ***u*** is given by the number of unemployed divided by the labour supply:

$$u_r = \frac{u}{l^s} \qquad [47]$$

3.6 Final demand for goods

Consumption

Consumption is determined by a model of intertemporal optimization. We present a simple version of this model. Suppose consumers maximize the following two-period inter-temporal problem:

$$argmax(c_1, c_2):\ \log c_1 + \frac{1}{1+\delta}\log c_2$$

[48]

$$subject\ to:\ c_1 + \frac{1}{1+r}c_2 = y_1^d + \frac{1}{1+r}y_2^{d,e} + wealth_0$$

where c_i and y_i^d denote real consumption and real disposable income in period *i,* for *i* = 1,2. The superscript *e* denotes expected value; the value of

y_2 is not known in the first period, so consumers have to form expectations about it; r is the real interest rate and d the personal discount rate. $Wealth_0$ denotes wealth accumulated from the past. The maximand is the present discounted value of inter-temporal utility. The appropriate discount rate is the personal discount rate d. The right-hand side of the budget constraint equals the present discounted value of resources and the left-hand side the present discounted value of consumption expenditure. The budget constraint says that consumers may save and borrow, but such that the present values of their incomes and expenditures remain equal to each other.

It is generally assumed that the personal discount rate d is equal to the real interest rate r. Under this condition, the first-order conditions for this problem imply

$$c_1 = c_2 . \quad [49]$$

So the consumption levels in both periods are equal to each other. This is the basic idea of consumption smoothing over time; consumers save and borrow to keep their consumption levels relatively constant.

By assuming an explicit formula for expected real disposable income in period 2, we can solve for c explicitly. We assume

$$y_2^{d,e} = y_1^d (1+g) ; \quad [50]$$

that is, income is assumed to grow at rate g. Then the formula for consumption becomes

$$c_1 = \left(1 + \frac{g}{2+r}\right) y_1^d + \left(\frac{wealth_0}{2+r}\right), \quad [51]$$

which indicates that the coefficient on current income is around 1 if g is around zero, and if the current level of income is at a normal level. The coefficient on wealth is around ½ in this model, but in a more general model with more periods, it is approximately equal to 1 divided by the number of periods. If the number of periods is very large, the coefficient equals $r/(1 + r) \approx r$ (r is the real interest rate).

Since reliable data on wealth are not available, we leave that variable out of the equation. The equation in the model (in log) is

$$c = y^d - r \,. \qquad [52]$$

Exports

Exports are determined by an interaction of foreign demand for Kenyan goods, which might be set as a function of the relative export price of Kenyan goods in shillings and the level of income of the trading partners (Y_N).

$$x = Y_N^{\ \beta_1} \left(\frac{e\, p_x}{p_d} \right)^{\beta_2} . \qquad [53]$$

The term in the bracket is the real exchange rate. P_d might be given by ($P^{m,s}$).

Equation 53 abstracts form quality effects or supply side effects. Such supply effects are crucial in developing countries such as Kenya (see Alemayehu 2002). The effects of the supply side could be modelled by adding the capital stock to the level equation, or the investment as a ratio of the capital stock or of value added to the percentage-change equation. The last term has indeed been highly significant in several studies of exports. Adding this effect, we get for the percentage change in exports

$$\hat{x} = \beta_1 \hat{Y}_N + \beta_2 (\,\hat{p}_x + \hat{e} - \hat{p}_d\,) + \phi \left(\frac{i}{y} \right)_{-1} . \qquad [54]$$

The income of the trading partners (Y_N) is exogenous.

3.7 The monetary block and the exchange rate

Money demand, money supply and the domestic nominal interest rate

The demand for money, M^d, is defined as a function of real GDP, the price level and the nominal interest. We have

$$M^d = \alpha Y + \beta P_y - \gamma i \qquad [55]$$

where M^d denotes nominal demand for money, Y nominal GDP, and i the nominal interest rates on bonds.

We assume that the exchange rate is floating, so that money supply is available as an exogenous policy instrument (semi-behavioural) given by $M^{S}*$. Equilibrium in the domestic money market then implies

$$i = \frac{1}{\gamma}\left(\alpha Y + \beta P_y - M^{S*}\right). \quad [56]$$

Given the practice of monetary policy in Kenya where the Central Bank directly targets inflation (see annex 2), equation 56 and the money supply (in first differences) can be written

$$\Delta i = \frac{1}{\gamma}\left[\alpha \Delta Y + \beta \Delta P_y - \Delta M^{S}\right] \quad [57]$$

$$\Delta M^{S} = \alpha \Delta Y + \beta \Delta P_y^{*} + \beta_1 \left[\Delta P_y - \Delta P_y^{*}\right] \quad [58]$$

with $\beta_1 < \beta$ where $P*$ is the target level of inflation.

This gives us the interest-rate equation of the model (see annex 2 for details):

$$\Delta i = \frac{\left(\beta - \beta_1\right)}{\gamma}\left[\Delta P_y - \Delta P_y^{*}\right]. \quad [59]$$

Exchange rate

The exchange rate is assumed to be floating. An increase in domestic interest rates makes domestic bonds relatively more attractive and causes the shilling to appreciate. Moreover, based on a Dornbusch-type analysis (Dornbusch 1976), the exchange rate may well overshoot its long-run value. This appreciation is assumed to take place relative to a steady-state rate of change of the exchange rate, which will reflect the difference between domestic and foreign inflation rates. It gives

$$\hat{e} = \alpha\left(\Delta i^{f} - \Delta i\right) + \beta\left(\Delta i^{f} - \Delta i\right) - \beta\left(\Delta i^{f} - \Delta i\right)_{-1} + (\hat{p}_d - \hat{p}_f)_{-1} \quad [60]$$

where i^f is the foreign interest rate, α is the coefficient for the basic response of the exchange rate to the change in the interest-rate differential, and β the coefficient for the overshooting part. The difference in the domestic and foreign inflation rates is lagged.

The real exchange is derived by definition

$$r\hat{e}r = \hat{e} + \hat{p}_f - \hat{p}_d \qquad [61]$$

where *rêr* is the real exchange rate (see annex 2 for details).

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Annex 1

Detailed derivation of major equations of the model

This annex is aimed at providing details of the model equation derivation so as to make the model accessible. This is important in particular for building capacity in policy application.

Consumption function

The utility function associated with consumption is given by

$$U = U\left(C_o, \ldots C_t \ldots C_T\right). \qquad [A1]$$

If we assume the underlining utility function is logarithmic we have

$$U\left(C\right) = \ln C . \qquad [A2]$$

The marginal utility (the first derivative) and the maximization condition (the second derivative) are given by

$$U'\left(C\right) = \frac{1}{C}$$
$$U''(C) = -\frac{1}{C^2} \qquad [A3]$$

For consumption function of period T (equation A4) and its constraint (equation A5), the formulation above can be given by

$$U = \ln Co + \frac{\ln C_1}{1+\delta} + \ldots\ldots + \frac{\ln C_t}{\left(1+\delta\right)^t} + \ldots\ldots + \frac{\ln C_T}{\left(1+\delta\right)^T} \qquad [A4]$$

$$Co + \frac{C_1}{1+r} + \ldots\ldots + \frac{C_T}{\left(1+r\right)^T} = Y_o + \frac{Y_1}{1+r} + \ldots\ldots + \frac{Y_T}{\left(1+r\right)^T} \qquad [A5]$$

Thus

$$\underset{C_t}{Max} \sum_0^T \frac{\ln C_t}{\left(1+\delta\right)^t} \text{ subject to } \sum_0^T \frac{\ln C_t}{\left(1+r\right)^t} = \sum_0^T \frac{\ln Y_t}{\left(1+r\right)^t} \qquad [A6]$$

Using Lagrange multiplier, this can be written as:

$$\underset{C_t,\lambda}{Max}\ L = \sum_0^T \frac{\ln C_t}{(1+\delta)^t} + \lambda\left[\sum_0^T \frac{\ln Y_t}{(1+r)^t} - \sum_0^T \frac{C_t}{(1+r)^t}\right]. \quad \text{[A7]}$$

The first-order conditions for the optimization are given as

$$\frac{\partial L}{\partial C_0} = \frac{1}{C_0} - \lambda = 0 \quad \text{[A8]}$$

$$\frac{\partial L}{\partial C_t} = \frac{1}{(1+\delta)^t}\cdot\frac{1}{C_t} - \frac{\lambda}{(1+r)^t} = 0 \quad \text{[A9]}$$

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•
•

$$\frac{\partial L}{\partial C_T} = \frac{1}{(1+\delta)^T}\cdot\frac{1}{C_t} - \frac{\lambda}{(1+r)^T} = 0 \quad \text{[A10]}$$

$$\frac{\partial L}{\partial \lambda_t} = \sum_0^T \frac{Y_t}{(1+r)^t} - \sum_0^T \frac{C_t}{(1+r)^t} = 0 \quad \text{[A11]}$$

Given the above formulation, let us compare consumption at two consecutive periods such as at time 0 and *t*. If we solve for λ in equations A8 and A9 and get the ratio of equations A9 to A8, we will have

$$\frac{C_t}{C_0} = \left(\frac{1+r}{1+\delta}\right)^t \quad \text{[A12]}$$

that can be generalized to

$$\frac{C_t}{C_{t-1}} = \left(\frac{1+r}{1+\delta}\right)^{\mathrm{t}} \text{ or } C_t = \left(\frac{1+r}{1+\delta}\right)^t C_{t-1}. \quad \text{[A13]}$$

If $r = \delta$ (that is, the personal discount rate equals the market rate), as is often the case, then $C_t = C_{t-1}$. This consumption smoothing is rooted in Friedman's classic work, *A Theory of the Consumption Function* (1957). The last two consumption-related equations, which are shown in the text, are derived based on the permanent income hypothesis explained below.

Imposing the assumption of permanent income hypothesis, say over two periods, on the intertemporal budget constraint (and hence having $C_1 + C_2/(1 + r) = Y_1 + Y_2/(1 + r)$) implies that one needs to find a value of permanent (average) income *Yp* such that the household would have the same intertemporal consumption possibilities in each period. This in turn implies that Y_p must satisfy the equality

$C_1 = Y_p$, which implies

$$Y_p + \frac{Y_p}{1+r} = W + Y_1 + \frac{Y_2}{1+r}. \quad \text{[A14]}$$

Equation A14 can be rewritten as

$$\frac{Y_p(1+r) + Y_p}{1+r} = W + Y_1 + \frac{Y_2}{1+r} \quad \text{[A15]}$$

$$= \frac{(2+r)}{(1+r)} Y_p = W + \left[Y_1 + \frac{Y_2}{1+r} \right] \quad \text{[A16]}$$

$$Y_p = \frac{(1+r)}{(2+r)} \left[Y_1 + \frac{Y(1+g)}{1+r} \right] + \left[\frac{1+r}{2+r} \right] W \quad \text{[A17]}$$

Using the definitions of adaptive expectation (Friedman 1957)

$Y_2 = (1 + g)Y_1$

$$= \left[\left(\frac{1+r}{2+r} \right) Y_1 + \left(\frac{1+g}{2+r} \right) Y_1 \right] + \left[\frac{1+r}{2+r} \right] W \quad \text{[A18]}$$

$$= \left[\left(\frac{(1+r)+(1+g)}{2+r} \right) Y_1 \right] + \left[\frac{1+r}{2+r} \right] W \quad \text{[A19]}$$

$$= \left[\frac{2+r}{2+r} + \frac{g}{2+r} \right] Y_1 + \left[\frac{1+r}{2+r} \right] W \quad \text{[A20]}$$

$$= \left[1 + \frac{g}{2+r} \right] Y_1 + \left[\frac{1+r}{2+r} \right] W \quad \text{[A21]}$$

This is the final equation used in the model. It was noted in the text that if the number of periods is very large, the coefficient for *W* equals $r/(1 + r) \approx r$. This is because if the term in the left side of equation A14 is set for *n* periods, it will appear as

$$Y_p + \frac{Y_p}{1+r} + \frac{Y_p}{(1+r)^2} + \frac{Y_p}{(1+r)^3} + \ldots + \frac{Y_p}{(1+r)^n}$$
$$\equiv Y_p\left(1 + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \ldots + \frac{1}{(1+r)^n}\right) \qquad \text{[A22]}$$

If we assume $1/(1 + r) = k$, the right-hand side of equation A22 can be written as an infinite geometric series that can be approximated by $1/(1- k)$, which equals to

$$\frac{1}{1-k} = \frac{1}{1-\left(\frac{1}{1+r}\right)} = \frac{1}{\frac{1+r-1}{1+r}} = \frac{r}{1+r}. \qquad \text{[A23]}$$

Input demand determination: investment goods and wage employment

As shown in the text, the inputs are determined in the context of a CES production function. We show the derivation of wage employment in this annex. The result can readily be replicated to determine the investment equation (demand for capital). Given the CES and the neoclassical condition for optimality, the cost-minimizing input combination, the demand for labour (hence wage employment) could readily be derived as follows.

Suppose the CES is given by

$$Y = \left[\beta_L L^{-\lambda} + \beta_K K^{-\lambda}\right]^{-1/\lambda}. \qquad \text{[A24]}$$

The optimal level of employment can be derived from the condition that the marginal product of labour should be equal to real wage. This is given as

$$MPL = \frac{\partial Y}{\partial L} = -1/\lambda \left[\beta_L L^{-\lambda} + \beta_K K^{-\lambda}\right]^{-1/\lambda - 1} \cdot \left(-\lambda L^{-\lambda-1}\beta_L\right) \qquad \text{[A25]}$$

$$= \frac{\beta_L\left[\beta_L L^{-\lambda} + \beta_K K^{-\lambda}\right]^{-1/\lambda - 1}}{L^{(1+\lambda)}}; \text{Note: } -1/\lambda - 1 \equiv \left(-1/\lambda\right)(1+\lambda) \qquad \text{[A26]}$$

$$= \frac{\beta_L\left[\beta_L L^{-\lambda} + \beta_K K^{-\lambda}\right]^{\left(-1/\lambda\right)(1+\lambda)}}{L^{(1+\lambda)}} \qquad \text{[A27]}$$

Thus,

$$\frac{\partial Y}{\partial L} = \beta_L \left(\frac{Y}{L}\right)^{1+\lambda} = \frac{w}{P} \quad \Leftarrow \text{using the neoclassical condition [A28]}$$

$$\beta\left(\frac{P}{w}\right)Y^{1+\lambda} = L^{1+\lambda} \qquad \text{[A29]}$$

$$L = \left[\beta_L\left(\frac{P}{w}\right)\cdot Y^{1+\lambda}\right]^{1+\lambda} \qquad \text{[A30]}$$

$$L = \beta_L^{1/1+\lambda}\left(\frac{P}{w}\right)^{1/1+\lambda} Y \qquad \text{[A31]}$$

$$L/Y = \beta_L^{\sigma}\left(\frac{w}{P}\right)^{-\sigma} \quad \text{or} \quad L = \beta_L^{\sigma}\left(\frac{w}{P}\right)^{-\sigma} Y; \text{ where } \quad \frac{1}{1+\lambda} \qquad \text{[A32]}$$

This last equation is basically the demand-for-labour equation. Similar procedure will also give the demand for investment. Thus, the latter is not done in this annex.

Price determination

In the model revenue is defined as

$$Revenue = P^f Z \qquad \text{[A33]}$$

where P^f is the price at factor cost and Z output.

Cost is assumed to behave and given as

$$C = c(Z). \qquad \text{[A34]}$$

Given equations A33 and A34, an inverse demand curve exists and is given by

$$P^f = P^f(Z). \qquad \text{[A35]}$$

The profit of the firm is thus given by

$$\pi = P^f Z - C(Z). \qquad \text{[A36]}$$

Given a profit tax of t_π, maximization of π entails setting the first derivative of the profit equation to zero. If the profit function is given by

$$\pi = (1 - t_\pi)\left[P^f Z - C(Z)\right], \text{ then} \quad [A37]$$

$$\frac{\partial \pi}{\partial Z} = (1 - t_\pi)MR - (1 - t_\pi)MC = 0$$
$$\text{Note: } \left(P^f Z\right)' = MR \text{ and } C'(Z) = MC \quad [A38]$$

Given the optimality condition that *MR = MC,* we can write this as

$$\frac{dR}{dZ} = MC \qquad \text{where: } \frac{dR}{dZ} \text{ is MR.} \quad [A39]$$

This implies the profit maximization condition based on equation A36 is derived as

$$P^f \cdot \frac{dZ}{dZ} + Z\frac{dP^f}{dZ} = MC \quad \Leftarrow \text{using the product rule} \quad [A40]$$

$$P^f\left(1 + \frac{dP^f}{dZ} \cdot \frac{Z}{P^f}\right) = MC \quad [A41]$$

$$P^f\left(1 + \frac{1}{\varepsilon}\right) = MC \quad [A42]$$

$$P^f = \left(1 + \frac{1}{\varepsilon}\right)^{-1} MC$$
$$= \left(1 - \frac{1}{\varepsilon}\right)^{-1} MC \quad [A43]$$

$\Leftarrow$ since the point elasticity of demand is normally negative.

Once we have equation A43, the steps from equation 2 to equation 4 in the text are fairly straightforward. Equation 4 in the main text shows that price is defined as a function of, *inter alia*, marginal cost *(MC)*. The *MC* is not observable, however. Three concepts are used. First, capacity is defined as the level of output with the minimum average long-run cost. *MC* at capacity equals the long-run average cost *(LAC)* since in the long run *MC = AC*. This is shown in figure A1.

Figure A1. Marginal cost *(MC)* equals the long-run average cost *(LAC)*.

The second concept used in deriving prices is that of duality. The idea is that in a mathematical programming context, if there is an objective function to be optimized (primal) its representation can be dual by interchanging the objective function and the constraints, and the solution will be similar.

The third and final concept used to derive the price equation is the Taylor expansion of the marginal cost around capacity output *(Z*)*. This basically says that it is possible to transform a function such as *f(Z)* into a polynomial form using the Taylor expansion of this function around *Z**. In such a procedure the coefficients of the various terms are expressed in terms of derivatives such as *f'(Z*)*, *f''(Z*)*, and so on, all evaluated at the point of expansion. Thus for a function *f(Z)* the Taylor expansion or approximation at *Z** is given by

$$f(Z) = f(Z^*) + \frac{1}{1!} f'(Z^*)(Z - Z^*) + \frac{1}{2!} f''(Z^*)(Z - Z^*)^2 + \ldots.. \qquad \text{[A44]}$$

If we limit such expansion to a first-degree polynomial (linear), this can be given as

$$f(Z) \approx f(Z^*) + f'(Z^*)(Z - Z^*) \qquad \text{[A45]}$$

If the function in equation A45 is given by *MC(Z)*, instead, we can rewrite equation A45 as

$$MC(Z) = MC(Z^*) + MC'(Z^*)(Z - Z^*) \qquad \text{[A46]}$$

$$At...Z^* \rightarrow MC(Z^*) = AC(Z^*)$$

$$MC(Z) = AC(Z^*) + \alpha Z^* \left(\frac{Z - Z^*}{Z^*} \right) \qquad \text{[A47]}$$

where α = *MC' at Z**, and the Z-Z*/Z* component after the plus sign (+) is ad hoc to get the *q* given below.

$$MC(Z) = AC(Z^*) + \alpha Z^* \left(\frac{Z}{Z^*} - 1 \right) \qquad \text{[A48]}$$

$MC(Z) = AC + \beta(q-1)$ where $\beta + \alpha Z^*$ and $q = Z/Z^*$.

This is basically the definition of *MC*, which is included in the price equation in the text (equation 5) and further worked out to arrive at the rest of the price equations (equation 9 and equations 12 to 17) that are given in the main text.

Wage determination

The wage determination in equations 24 to 26 in the text is arrived at using the Nash bargaining solution to the wage negotiation process. This is done by optimizing the equation given in equation 24 in the text. Equation 24 can be stated as follows:

$$\underset{w}{Max}... \left(\frac{p_y hl - w(1+s_f)l}{p_c} \right)^{\alpha} \left(\frac{w(1-s_l)}{P_c} - F \right)^{1-\alpha} \qquad \text{[A49]}$$

Thus we are basically maximizing A49 with respect to wage *w*. We use the product and composite function rules. First differentiate the first part of the product and multiply this by the second part and then differentiate the second part and multiply the result by the first part. Then we set the whole to zero—first-order condition. We get

$$-\alpha\left(\frac{(1+s_f)l}{p_c}\right).\left(\frac{p_y hl-w(1+s_f)l}{p_c}\right)^{\alpha-1}\left(\frac{w(1-s_l)}{P_c}-F\right)^{1-\alpha}+$$
$$(1-\alpha)\frac{(1-s_l)}{p_c}\left(\frac{w(1-s_l)}{P_c}-F\right)^{1-\alpha-1}\left(\frac{p_y hl-w(1+s_f)l}{p_c}\right)^{\alpha}=0 \quad [A50]$$

Let

$$X=\left(\frac{p_y hl-w(1+s_f)l}{p_c}\right)$$
$$Y=\left(\frac{w(1-s_l)}{P_c}-F\right) \quad [A51]$$

Using A51 and rearranging, equation A50 can be written as

$$\alpha\left(\frac{(1+s_f)l}{p_c}\right)X^{\alpha-1}Y^{1-\alpha}=(1-\alpha)\frac{(1-s_f)}{p_c}X^{\alpha}Y^{-\alpha}. \quad [A52]$$

Multiplying [A52] through by $X^{1-\alpha}Y^{\alpha}\left(\frac{p_c}{(1+s_f)}\right)$ results in

$$\alpha lY=(1-\alpha)\frac{(1-s_f)}{(1+s_f)}X \quad [A53]$$

The definition of X in [A51] implies

$$X=\left(\frac{p_y hl-w(1+s_f)l}{P_c}\right)=\frac{p_y l}{p_c}\left(h-\frac{w(1+s_f)}{p_y}\right) \quad [A54]$$

Product and consumption wage are defined in equations 20 and 21 in the main text, reproduced here for convenience:

$$w_y=\frac{w(1+s_f)}{p_y}$$
$$w_c=\frac{w(1-s_l)}{p_c} \quad [A55]$$

Combining A51, A53 and A55 we could get

$$X = \frac{p_y l}{p_c}\left(h - w_y\right)$$
$$Y = w_c - F \qquad \text{[A56]}$$

Substituting for *X* and *Y* in A53 from A56, we get

$$\alpha l(w_c - F) = (1-\alpha)\frac{(1-s_l)}{(1+s_f)}\frac{p_y l}{p_c}(h - w_y). \qquad \text{[A57]}$$

Multiplying A57 through by $\dfrac{(1+s_f)p_c}{(1-s_l)p_y}$ will give

$$\alpha\left(\frac{(1+s_f)}{(1-s_l)}\frac{p_c}{p_y}\right)\left(w_c - F\right) = \left(1-\alpha\right)\left(h - w_y\right). \qquad \text{[A58]}$$

Given the definitions of the wedge, denoted Λ, in equation 22 of the main text,

$$\Lambda = \frac{w_y}{w_c} = \frac{(1+s_f)}{(1-s_l)}\frac{p_c}{p_y},$$

equation A58 can be written as

$$\alpha\Lambda w_c - \alpha\Lambda F = \left(1-\alpha\right)h - \left(1-\alpha\right)w_y \qquad \text{[A59]}$$

Given the definition of Λ, w_c, and w_y

$$\Lambda w_c = \left(\frac{(1+s_f)}{(1-s_l)}\frac{p_c}{p_y}\right)\left(\frac{w(1-s_l)}{p_c}\right) = \frac{w(1+s_f)}{p_y} = w_y \qquad \text{[A60]}$$

Substituting A60 into A59 we get

$$\alpha w_y - \alpha\Lambda F = \left(1-\alpha\right)h - \left(1-\alpha\right)w_y \qquad \text{[A61]}$$

Rearranging A61 gives the desired result (shown as equation 25 in the main text):

$$w_y = \left(1-\alpha\right)h + \alpha\Lambda F \qquad \text{[A62]}$$

The next issue is what is *F,* the fallback position? Conceptually, it is what workers get when they lose their jobs. Here the analysis gets a little less formal. We consider three components for the fallback position: the possibility of getting another formal job, unemployment and welfare benefits, and the possibility of finding self-employment in the informal sector. The last seems the most logical in the Kenyan context.

We assume that an unemployed worker tries to get another job in the formal sector. We assume that with probability p the worker succeeds and thus will keep on getting the normal consumption wage w_c. With probability $(1 - p)$ the worker does not find another formal sector job and so may get benefits in the form of formal unemployment or welfare benefits B. Define the replacement rate rp as the ratio of B to the consumption wage: $rp = B/w_c$, so that $B = rp\ w_c$. The replacement rate indicates the degree to which the social benefits replace the wage loss if workers lose their job.

In addition to getting benefits, a worker who has lost a job may find self-employment in the informal market. We assume that productivity in the informal sector is a faction φ of the productivity in the formal sector, with φ < 1. However, since the informal sector is not taxed, and social security contributions or pension premiums do not exist, workers in the informal market get to keep all their productivity. So the worker's income in the informal market equals *h,* and we get for *F*

$$F = pw_c + (1-p)(rpw_c + \phi h)\,. \qquad \text{[A63]}$$

It makes sense that the probability of finding a job p is negatively related to the unemployment rate ur. We assume that $p = 1 - ur$. Then if $ur = 0$, $p = 1$, which means that getting a new job is virtually certain. Substituting gives

$$\begin{aligned} F &= (1-ur)w_c + ur(rpw_c + \phi h) = (1-ur+ur.rp)w_c)ur.\phi h \\ &= (1-ur(1-rp))w_c + ur.\phi h \end{aligned} \qquad \text{[A64]}$$

For countries with a well-developed social security system, the replacement rate is a major determinant of wage negotiations. The higher the replacement rate, the better the fallback position of workers. For Kenya, the replacement rate is probably close to zero as welfare benefits are virtually non-existent and unemployment is very high. Then *F* simplifies to

$$F = (1 - ur)w_c + ur.\phi h \,. \qquad \text{[A65]}$$

So if the unemployment rate is high, informal sector self-employment becomes the dominant part of the fallback position of workers. If *ur* is low, getting another job becomes more relevant. We now substitute the equation for *F* into equation A62 to obtain the equation presented in the text. First, as an intermediary step we multiply *F* by Λ and, using the fact that Λ = *wy* / *wc*

$$F\Lambda = (1 - ur)w_y + ur.\phi h\Lambda \qquad \text{[A66]}$$

Substituting this into equation A62 gives

$$w_y = \frac{1 - \alpha + \alpha ur.\phi\Lambda}{1 - \alpha(1 - ur)} h \qquad \text{[A67]}$$

So w_y is proportional to productivity ***h*** and depends negatively on the unemployment rate *ur* and positively on the wedge.[6]

Since $w_y = w(1 + s_f)/p_y$, we get

$$\log w(1 + s_f) = \log p_y + \log h + \beta_1 \log \Lambda - \beta_2 ur \qquad \text{[A68]}$$

The specific form of the linearization is not obvious. We entered the wedge log-linearly and the unemployment rate linearly. This is in line with empirical practice, but other specifications could have been chosen as well. This is equation 26 in the main text.

Equation 27 takes first differences of this equation. Taken straightforwardly and using the definition of Λ given in equation 22 of the main text, this ~~gives[HvH1]~~solves to:[HvH2]

[6] The sign of *ur* is ambiguous in the above formula, but as long as formal sector jobs are more attractive than informal employment, *ur* will have a negative effect on wages. This can be seen by the above equation for *F*: if $w_y > h$, then *F* depends negatively on *ur:* The fallback position falls if the chance of getting another formal job falls. Equation A62 then says that a lower fallback position *F* reduces wages.

$$\Delta \log w = -\Delta \log(1 + s_f) + \Delta \log p_y + \Delta \log h$$
$$+ \beta_1 \begin{bmatrix} \Delta \log p_c - \Delta \log p_y + \Delta \log(1 + s_f) \\ - \Delta \log(1 + s_l) \end{bmatrix} - \beta_2 \Delta ur - \beta_3 ur$$

There are two steps to get to equation 27 in the text. First, note that log(1 + *x*) = *x*/(1 + *x*), so that approximately, log(1 + *x*) = *x*/(1 + *x*), and log(1 – *x*) = – *x*/(1 – *x*). Second, the level term $\beta_3 ur$ is added. This may be justified because ***ur*** enters the theoretical equation in a highly non-linear way. It also is often an important empirical term in a wage equation (basically the original Phillips curve term).

Annex 2

Note on the money market block

The money market and interest rate

The money market block consists of three equations: a behavioural equation for the demand for money, a policy (semi-behavioural) equation for the money supply, and a market-clearing equation stating that money demand equals money supply. This last equation may also be thought of as the interest-rate equation, since it is the interest rate that clears the market in a liberalized money market. Formally we have

$$\ln M^d = \alpha \ln Y + \beta \ln P_y - \gamma i \qquad \text{[A2-1]}$$

$$\ln M^S = PR \qquad \text{[A2-2]}$$

$$\ln M^d = \ln M^S \qquad \text{[A2-3]}$$

where M^d denotes money demand, M^S money supply, *Y* real GDP, *P* the price level (price of final demand, for instance the consumer price index (CPI), *i* the nominal interest rate, *PR* a semi-behavioural equation (policy rule) and ln natural logarithm.

By combining equations A2-1 and A2-3 we can solve for the interest rate that clears the money market for given levels of money supply:

$$i = \frac{1}{\gamma}\left[\alpha \ln Y + \beta \ln P - \ln M^{S}\right]. \qquad \text{[A2-4]}$$

Equations A2-2 and A2-4 are probably the easiest way to model the money market in the model. In equation A2-2 we specify the money supply as set by the Central Bank, and equation A2-4 then gives the result for the interest rate.

The most common policy options for setting the money supply are targeting the money supply, targeting the interest rate and targeting the inflation rate. Under the first rule, M^S is set equal to M^{S*}, where * denotes the target. The resulting interest rate is given by equation A2-4, with M^S replaced by M^{S*}. In reduced form the money block in this case is

$$M^{S} = M^{S*} \qquad \text{[A2-5]}$$

$$i = \frac{1}{\gamma}\left[\alpha \ln Y + \beta \ln P_y - \ln M^{S*}\right] \qquad \text{[A2-6]}$$

If the Central Bank targets the interest rate, the money supply rule and reduced-form interest-rate equations are given by

$$\ln M^{S} = \alpha \ln Y + \beta \ln P_y - \gamma i^{*} \qquad \text{[A2-7]}$$

$$i = i^{*}. \qquad \text{[A2-8]}$$

If the Central Bank targets the inflation rate, a variety of policy rules will over time achieve the objective. A popular set of policy rules and the resulting reduced-form equations for the interest rate are given by

$$\ln M^{S} = \alpha \ln Y + \beta \ln P_y^{*} + \beta_1\left[\ln P_y - \ln P_y^{*}\right] \quad \text{with } \beta_1 < \beta \qquad \text{[A2-9]}$$

$$i = \frac{(\beta - \beta_1)}{\gamma}\left[\ln P_y - \ln P_y^{*}\right] \qquad \text{[A2-10]}$$

Equation A2-9 indicates that the money supply fully accommodates an increase in money demand resulting from increased real economic growth, but it only partially accommodates an increase in money demand resulting from inflation. The accommodation parameter for inflation is $\beta 1$, with $\beta_1 < \beta$. If $\beta_1 = 0$, inflation is not accommodated at all.

In first differences, the reduced-form equation for the interest rate in this case is:

$$\Delta i = \frac{(\beta - \beta_1)}{\gamma}\left[\ln \Delta P_y - \ln \Delta P_y^{\,*}\right] \qquad \text{[A2-11]}$$

where Δ indicates first difference. So, this money supply rule implies that the interest rate rises if inflation is above its target. The speed with which this happens depends negatively on β_1. Thus β_1 may be seen as an indicator of the aggressiveness with which the Central Bank fights inflation. A low value of β_1 implies that the Central Bank accommodates inflation little, and therefore that the interest rate rises a lot if inflation is above its target.

In its publication *The Practice of Monetary Policy in Kenya, 2000,* the Central Bank of Kenya states that it currently targets the money supply as an intermediary target for controlling inflation. However, on page 80 it writes 'against the background of substantive liberalization of Kenya's financial markets, and the gradual integration of Kenya in the very volatile global financial environment, it will soon become extremely difficult to put meaningful quantitative values to the major financial aggregates that will produce the desired results for inflation. This will therefore necessitate the Central Bank to target the rate of inflation directly, instead of setting guidelines for intermediate objectives such as the money supply.'

So the shift from money targeting to inflation targeting is only a pragmatic change in method, not a change in objective. From a modelling point of view, we may therefore assume that inflation targeting has been the option chosen by the Central Bank for the recent past and for the future.

The approach followed is therefore to use the following two equations to describe the money market in the model in its structural form

$$\Delta i = \frac{1}{\gamma}\left[\alpha \Delta \ln Y + \beta \Delta \ln P_y - \Delta \ln M^S\right] \qquad \text{[A2-12]}$$

$$\Delta \ln M^S = \alpha\Delta \ln Y + \beta\Delta \ln P_y^* + \beta_1\left[\Delta \ln P_y - \Delta \ln P_y^*\right] \quad \text{with } \beta_1 < \beta \qquad [A2\text{-}13]$$

The first equation is the structural equation for the interest rate and the second the policy rule for money supply. Together, they imply the following reduced-form equation for the interest rate:

$$\Delta i = \frac{(\beta - \beta_1)}{\gamma}\left[\Delta \ln P_y - \Delta \ln P_y^*\right]. \qquad [A2\text{-}14]$$

It is advisable to use the structural form, equations 12 and 13 in the model, because then we can also analyse the effects of a change in the money supply by simply changing equation 13. This would not be possible if we use the reduced-form equation 14 in the model.

Estimation and remark

Equation A2-12 may be estimated directly or by inverting the estimated money-demand function A2-1. The policy rule A2-13 may be estimated directly after we have information about P^*, or by making an assumption about it. One assumption is that P^* is constant, in which case ln P^* becomes part of the constant term and dln $P^* = 0$. Another assumption is that ln P^* moves with ln P, in which case ln P^* will be subsumed in the ln P term.

If we model the money market in its structural form, care should be taken that it remains consistent with the effects on money supply resulting from other parts of the model. For instance, it implies that any inflow of foreign capital is sterilized and that the domestic financing of the deficit is in line with the money-supply creation set by the money-supply rule A2-13.

Exchange rate in the model

Apart form the Dornbush type specification described in the main text, the following set of exchange rate models is also experimented with in the estimation stage. Thus, this part of the annex is aimed at providing alternative forms of exchange rate models that could be explored for improving the exchange rate block of the model. Work along this line is in progress (see Were et al. 2001).

Alternative specifications

FLEXIBLE-PRICE MONETARY MODEL

This model assumes that, first, the PPP continuously holds; thus

$$S_t = P_t - P^*_t.$$

Second, these prices must be consistent with the money market equilibrium, which implies

$$M_t = P_t + \phi Y - \lambda r_t \quad \text{and} \quad M^*_t = P^*_t + \phi Y^* - \lambda r^*_t \qquad \text{[A2-15]}$$

This implies

$$S_t = M_t - M^*_t - \phi Y + \phi^* Y^* + \lambda r_t - \lambda^* r^*_t + \varepsilon_t \qquad \text{[A2-16]}$$

Assuming identical money-demand functions

$$S_t = (M_t - M^*_t) - \phi(Y - Y^*) + \lambda(r_t - r^*_t) + \varepsilon_t \qquad \text{[A2-17]}$$

An estimable version of this can be derived by assuming that the money-demand equation can be depicted by price differential.

STICKY-PRICE MONETARY MODELS

In Dornbusch (1976) the assumption that the PPP continuously holds is left out. Frankel (1979) gives an empirical variant of this model:

$$S_t = (M_t - M^*_t) - \phi(Y - Y^*) - \alpha(r_t - r^*_t) + \beta(\rho - \rho^*_t)\varepsilon_t \qquad \text{[A2-18]}$$

where ρ denotes expectation held at time t about the long-run rate of inflation. When Frankel's suggestion is included in the original Dornbusch model, it implies that the gap between the current and the equilibrium level of exchange rate is proportional to real interest-rate differential,[7] which implies

[7] Note that in the theoretical mode of Dornbusch $r = r^* + S^e$, where S^e is the expected rate of change in S and can be defined as $S^e = -\eta(S - \overline{S})$ where η is an expectation parameter S (bar) of the equilibrium level of exchange-rate parameter.

$$S - \overline{S} = \left(- \frac{1}{\eta} \right) \left[(r - r^*) - (\rho - \rho^*) \right]. \qquad \text{[A2-19]}$$

If this is combined with a long-run PPP assumption, then

$$\overline{S}_t = \overline{P} - \overline{P}^* = (M_t - M^*_t) - \phi(Y - Y^*) + \lambda(r_t - r^*_t) + \varepsilon_t \qquad \text{[A2-20]}$$

Having the above two equations substituting A2-20 in A2-19 and the long-run condition that

$\overline{r} - \overline{r}^* = \rho - \rho^*$ and where $\alpha = 1/\eta$ and $\beta = \lambda + (1/\eta)$

gives the Frankel formulation (equation A2-18). In empirical analysis, long-run interest rates are used as a proxy for the long-run expected inflation. Note also that if the sticky-price model works we expect a negative coefficient for the interest-rate differential. If the flexible-price model works, on the other hand, we expect a positive sign. The significance of the long-run interest-rate differential may also suggest whether the sticky-price model works or not.

THE PORTFOLIO BALANCE MODEL

This model modifies the sticky-price model by introducing cumulative trade balance or current account balance as an additional variable (Meese and Rogoff 1983a, b; Isard 1995)

$$S_t = a_0 + a_1(M_t - M^*_t) + a_2(Y - Y^*) + a_3(r_t - r^*_t) + a_4(\rho - \rho^*_t) + a_5 \int TB + a_6 \int TB^* + \varepsilon_t \qquad \text{[A2-21]}$$

The trade imbalance is believed to show redistribution of wealth internationally, which in turn affects each country's income and expenditure. This will have an effect on the real exchange rate, which needs to be consistent with the long-run current account balance. The above model is sometimes referred as the sticky-price hybrid model. It is a general model, which, upon appropriate restrictions, can yield the two other models.

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