

Are Prior Restrictions on Factor Shares Appropriate in Economic Growth Accounting Estimations?

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Abstract

Several studies make different prior assumptions on the magnitude of factor shares and scale of production when accounting for economic growth. The initial Solow estimations, for instance, assumed a capital share of 0.3 and constant returns to scale. Most authors have subsequently used the same restrictions just because they were used in previous studies, even when production in the countries under study may not necessarily be taking place under constant returns to scale, and capital share may be a value not any close to 0.3. This is likely to distort growth accounting estimation results. This study investigates whether these prior restrictions on factor shares and scale of production as commonly used in the literature are appropriate. The paper also examines whether there is any change in the explanatory power of the model when the Solow Model is augmented with human capital accumulation. An improvement in the explanatory power of the model after augmentation may be indicative of misspecification of the classical Solow model. Policy advice from a misspecified model results may, therefore, be misleading. Using Kenyan data and structural vector autoregressions, the main observations from the results are: first, in all cases of the unrestricted estimations, the share of physical capital is less than 0.1, which is less than the commonly used 0.3. An estimation that imposes 0.3 as the share of physical capital in this case would therefore not be in line with the data generating process, leading to biased results. Secondly, in all cases, the explanatory power of the model decreases when restrictions on factor shares are imposed. The findings also show that augmenting the Solow model with human capital accumulation improves the explanatory power of the model. In addition, the results show that restrictions on factor shares grossly underestimate the contribution of the factors to economic growth, while exaggerating the contribution of own shocks.

Acronyms

CBK	Central Bank of Kenya
DGP	Data Generating Process
IFS	International Financial Statistics
SVAR	Structural Vector Auto regressions

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1. Background

When empirically accounting for sources of economic growth in different countries, different authors have used different formulations of growth models. The most commonly used is the Solow model, which assumes two factors of production: labour-augmenting technical progress, and constant returns to scale (Solow, 1956). Other models have augmented the Solow model with human capital (Mankiew, Romer and Weil, 1992), with the explanation that human capital is an important factor of production that needs to be considered in economic growth models. In the process of accounting for growth, most studies impose prior restriction on factor share coefficients and assume that factor shares are some specific values. Solow (1956), for instance, assumed that the share of physical capital in output in the US at the time of his study was 35 per cent, and that production was taking place under constant returns to scale. Surprisingly, these assumptions have been adopted blindly by most authors undertaking empirical growth accounting studies just because Solow (1956) assumed constant returns to scale and a capital share of 0.35 (Gollin, 1996 and Klenov and Rodriguez, 1997). These assumptions, in most cases, may not be supported by the data generating process (DGP) in the particular countries under study.

While the assumption of 0.35 on physical capital share could have been true for the US economy in the 1950s when Solow's work was done, it may not have been the case in the 1990s and 2000s. Worse still, it may not have been true for other countries even at the time Solow's work was done in the 1950s and now. The assumption that production takes place under constant returns to scale may have been true in the US economy in the 1950s, but not necessarily in the 1990s or now and definitely may not be the case in other countries. Other studies have gone ahead and not only assumed physical capital share coefficient to be 0.3, but also used these coefficient restrictions to carry out cross-country comparisons, with the assumption that factor shares are the same for all countries in their sample (Gollin, 1996 and Klenov and Rodriguez, 1997). Other studies have used these *ad hoc* coefficient restrictions as a basis for theoretical coefficient expectations; that physical capital share for instance is "theoretically" expected to be 0.3 (Mankiew, Romer and Weil, 1992). There are several issues that are questionable with such restrictions. First, the assumption that capital share can be the same in all countries is misleading and may bias the results. Secondly, assigning a country's factor share any particular value without any logical reason

to support that value just because it is common practice to assign that particular value to that factor shares when in reality the share of that factor in the particular country's output may be different will again bias the results. Third, assuming constant returns to scale in production for any country without any reason to believe that production in that country takes place under constant returns would be misleading if production takes place under either increasing or decreasing returns to scale. Most studies contend that any slight change in assumptions about the size of the factor shares and returns to scale can lead to very significant differences in growth accounting results, leading to wrong policy advice (Klenov and Rodriguez, 1997 and World Bank, 2000).

Even with the apparent shortcomings of prior coefficient restrictions, most authors have continued to use them without interrogating their appropriateness. This study uses Kenyan data to argue that such coefficient restrictions are inappropriate and will lead to misleading results and policy advice. It argues that the determination of these factor shares and whether production takes place under constant, increasing or decreasing returns to scale should be determined by the data generating process. We start with a restricted model where we impose 0.3 as the capital share and constant returns to scale in line with Solow (1956). We then compare the results with unrestricted models, where the parameters are free and the factor shares determined by the data generating process using the econometrics approach. We examine whether the explanatory power and model performance of the unrestricted model is any different from the restricted model. We then augment the Solow model and compare the results of the augmented version of the unrestricted Solow model with the results of the unrestricted original Solow formulation. The augmentation is done by including human capital accumulation as one of the explanatory variables in the Solow model. This is done to determine whether the augmented Solow formulation with three factors: human capital, physical capital and labour is more appropriate in explaining growth than the original Solow formulation with only two factors: physical capital and labour, as argued by Mankiew, Romer and Weil (1992). If the augmented model has a higher explanatory power than the original Solow model, then we can interpret this to mean that the original Solow model is insufficient in explaining growth for policy purposes (is misspecified).

The results show that prior ad hoc restrictions underestimate the contribution of factor growths in economic growth, while magnifying

the contribution of own shocks. The results also show that the restricted Solow model is in fact misspecified. The unrestricted Solow model returns a higher explanatory power than its restricted counterpart. We interpret these results to imply that the exact values of the factor shares are best determined by the data generating process and not just the ad hoc assumptions. In addition, the augmented Solow model has found a higher explanatory power than the original Solow formulation, implying that human capital is an important factor in explaining growth, and needs to be taken into account in growth model formulations. This result is in line with the findings of Mankiew, Romer and Weil (1992), that augmenting the Solow model with human capital increases the explanatory power of the model. The results, however, find no support for their argument that the augmentation increases the impact of physical capital on income. In fact, our findings show that the impact of physical capital on output decreases with the augmentation.

1.1 Organization of the Paper

The rest of the paper is organized as follows: Section two reviews the various approaches that have been used to estimate factor shares; Section three details the empirical strategy adopted, and the empirical results; while Section four summarises and concludes the study.

2. Approaches to the Estimation of Factor Shares

2.1 Factor Share Estimates from Simple Cobb-Douglas Technology (Models 1 and 2)

Assuming a Cobb-Douglas technology of the form:

$$Y_t = Ae^{\beta t} K^\alpha L^{1-\alpha} \dots\dots\dots(1)$$

where Y is output, K is capital stock, L is total employment, and the expression $Ae^{\beta t}$ is the total factor productivity (TFP), α is the capital share in total output and $(1-\alpha)$ is the relative share of labour in total output. The fixed component of TFP A is assumed to grow at the rate β . To transform the production function into the intensive form, we divide by the labour units and take the natural logs of the intensive form production function to get:

$$\ln y_t = \ln \alpha_t + \beta \ln t_t + \alpha \ln k_t \dots\dots\dots(2)$$

where $\ln y_t$ is the natural logarithm of the output-labour ratio, $\ln k_t$ is the natural logarithm of the capital-labour ratio and $\ln \alpha$ is the natural log of A/L . Since both A and α are not observable, we obtain the estimate of α as the anti-log of the residuals from the regression (2). The parameter estimate α gives the share of capital in output and the share of labour in output is given by $(1-\alpha)$. The results from the estimation of equation (2) are given in the sub-section (3.4.1). Another specification of (1) where the Cobb-Douglas technology is linearized in its extensive form is:

$$\ln \tilde{y}_t = \tilde{a}_t + \beta \ln \tilde{t} + \alpha \ln \tilde{k}_t + \delta \ln \tilde{l}_t \dots\dots\dots(3)$$

where \tilde{a}_t are the residuals (the Solow residual), \tilde{y} is nominal GDP, \tilde{k}_t is the capital stock and \tilde{l}_t are the labour units. The results from the estimation of equation (3) are given in sub-section (3.4.2).

2.2 Calculating Factor Shares from the Solow Model (Model 3)

Solow (1956) decomposes growth in output into growth in the factors of production (physical, capital and labour), and the growth of the efficiency in the utilization of these factors. Solow considered a simple model with two factors of production and labour-augmenting technology over time of the form:

$$Y(t) = [K(t)]^\alpha [A(t)L(t)]^{1-\alpha} \dots\dots\dots(4)$$

where $Y(t)$ represents the total output in the economy in time t , $K(t)$ represents capital stock in the economy in time t , $L(t)$ represents the

total labour force in the economy in time t and $A(t)$ represents labour efficiency in the economy in time t . To measure the change in output, equation (2.4) is differentiated with respect to t so that;

$$\frac{\partial Y}{\partial t} = \frac{\partial Y}{\partial K} \frac{\partial K}{\partial t} + \frac{\partial Y}{\partial L} \frac{\partial L}{\partial t} + \frac{\partial Y}{\partial A} \frac{\partial A}{\partial t} \dots\dots\dots(5)$$

From equation (4);

$$\frac{\partial Y}{\partial K} = \alpha [K(t)]^{\alpha-1} [A(t)K(t)]^{1-\alpha} = \frac{\alpha Y}{[K(t)]}, \quad \frac{\partial Y}{\partial L} = \frac{(1-\alpha)Y}{[L(t)]} \text{ and}$$

$$\frac{\partial Y}{\partial A} = \frac{(1-\alpha)Y}{[A(t)]}$$

Therefore,

$$\frac{\partial Y}{\partial t} = \frac{\alpha Y}{[K(t)]} \cdot \frac{\partial K}{\partial t} + \frac{(1-\alpha)Y}{[L(t)]} \cdot \frac{\partial L}{\partial t} + \frac{(1-\alpha)Y}{[A(t)]} \frac{\partial A}{\partial t} \dots\dots\dots(6)$$

The growth factor in the economy is a proportion of the output in the previous period obtained by dividing both sides of equation (6) by Y so that;

$$\frac{\partial Y}{\partial t} \frac{1}{Y} = \frac{\alpha Y}{[K(t)]} \cdot \frac{\partial K}{\partial t} \frac{1}{Y} + \frac{(1-\alpha)Y}{[L(t)]} \cdot \frac{\partial L}{\partial t} \frac{1}{Y} + \frac{(1-\alpha)Y}{[A(t)]} \frac{\partial A}{\partial t} \frac{1}{Y} \dots\dots\dots(7)$$

$$\Rightarrow \frac{\partial Y/\partial t}{Y} = \alpha \frac{\partial K/\partial t}{[K(t)]} + (1-\alpha) \frac{\partial L/\partial t}{[L(t)]} + (1-\alpha) \frac{\partial A/\partial t}{[A(t)]}$$

The term on the left of equation (7) is the proportional change in output. The first two terms on the right are the proportional change in capital stock and labour, respectively. The remaining term on the right is the Solow residual and gives the effects of productivity improvements on GDP. The estimable form of equation (7) can be re-written as:

$$\dot{y}_t = \alpha \dot{k}_t + \lambda \dot{l}_t + \dot{a}_t$$

where $\dot{y}_t = \frac{\partial Y_t/\partial t}{Y_t}$, $\dot{k}_t = \frac{\partial K_t/\partial t}{K_t}$, $\dot{l}_t = \frac{\partial L_t/\partial t}{L_t}$ and $\dot{a}_t = \frac{\partial A_t/\partial t}{A_t}$
(8)

are the growth rates of output, physical capital stock, labour and total factor productivity respectively. The coefficients α and λ are the parameter estimates from the estimation of (8) and gives the factor shares in output. TFP growth given by \dot{a}_t is obtained as the residual from the estimation of (8). The variables needed to carry out estimation of (8) are therefore GDP growth, physical capital growth and labour

growth. The results from the estimation of equation (8) are given in the sub-section (3.4.3).

2.3 Calculating Factor Shares from the Augmented Solow Model (Model 4)

Mankiew, Romer and Weil (1992) argue that human capital accumulation is a major factor of production whose contribution to economic growth needs to be accounted for separately from the contribution of physical capital. Solow (1956) does not make this separation and assumes that there is only one type of capital in production; physical capital. The augmentation of the Solow model with human capital results into a variant of the original Solow formulation of the form:

$$Y_t = A_t e^{\beta t} K_t^\alpha H_t^\delta L_t^\lambda \dots\dots\dots(9)$$

where H is the stock of human capital and δ is the share of human capital in output. $\alpha + \delta + \lambda = 1$ if production takes place under constant returns to scale. The formulation in equation (9) implies that income growth can be calculated as:

$$\frac{\partial y / \partial t}{Y} = \left[\alpha \frac{\partial K / \partial t}{K(t)} + \lambda \frac{\partial L / \partial t}{L(t)} + \delta \frac{\partial H / \partial t}{H(t)} + \frac{\partial A / \partial t}{A(t)} \right] \dots\dots\dots(10)$$

where α, δ and λ are the factor shares in output. The estimable form of equation (10) is of the form:

$$\dot{y}_t = \alpha \dot{k}_t + \lambda \dot{l}_t + \delta \dot{h}_t + \dot{a}_t \dots\dots\dots(11)$$

where $\dot{a}_t = \frac{\partial A_t / \partial t}{A_t}$

$$\dot{y}_t = \frac{\partial Y_t / \partial t}{Y_t}, \dot{k}_t = \frac{\partial K_t / \partial t}{K_t}, \dot{l}_t = \frac{\partial L_t / \partial t}{L_t}, \dot{h}_t = \frac{\partial H_t / \partial t}{H_t} \text{ and}$$

are the growth rates of output, physical capital stock, labour, human capital and total factor productivity, respectively. The coefficient estimates of α, δ and λ are the factor shares in output. TFP growth given by \dot{a}_t is obtained as the residual from the estimation of (11). Therefore, the variables needed to carry out estimation of (11) are GDP growth, physical capital growth, labour growth and human capital growth. The results from the estimation of equation (11) are given in the sub-section (3.4.4). In the next section, we carry out empirical estimation of the unrestricted models reviewed in section two and the results compared with their respective restricted counterparts assuming that the physical capital share is 0.3.

3. Empirical Strategy

The first task in the estimation process would be to get data on the variables of interest. Unit root tests are then conducted on the variables to determine their order of integration. Different approaches reviewed in section two to estimate factor shares and compare the restricted and the unrestricted model estimates and model performance are used. The different estimates are then used to calculate TFP growth and to account for sources of economic growth in Kenya. The growth accounting outcomes from both the restricted and the unrestricted models are then compared for any differences.

3.1 Data

While GDP, labour and human capital are all available from published data sources, physical capital stock is not available and, therefore, must be estimated from the stock of investments. This is done using the perpetual inventories approach. We also collect annual time series data on nominal GDP, labour and human capital between 1982 and 2006. We measure human capital accumulation as the total enrolment in higher learning institutions, and labour is the total workforce. The different data sources include: Central Bank of Kenya (CBK) Quarterly Bulletin, Monthly Economic Reviews, International Financial Statistics (IFS), 2007 CD ROM and World Bank Africa database 2007 CD ROM. The first step in data consolidation is to calculate the stock of physical capital.

3.2 Estimates of Physical Capital Stock

To calculate physical capital stock, we use the perpetual inventory method which argues that the stock of physical capital is the accumulation of the stream of past investments, that is:

$$K_t = I_t + (1 - \delta)K_0 \dots\dots\dots(12)$$

where K_t is the stock of physical capital in period t , δ is the rate of depreciation of physical capital, K_0 is the initial physical capital stock and I_t is the investment in period t . Initial capital is calculated following the formulation given by Park (1995) as:

$$K_0 = \frac{I_0(1+g)}{g+\delta} \dots\dots\dots(13)$$

where g is the historical average of the growth rate of investments. The value of initial physical capital obtained from equation (13) is substituted into equation (12) to generate the series of physical capital stock for the whole of the sample period. Physical capital growth is then obtained from the calculated level series. Figures 3.1(a) and (b) show the evolution of physical capital stock and capital growth in Kenya respectively, from 1982 to 2006 as calculated using the perpetual inventory approach.

Figure (1a) shows that the general trend of physical capital was positive over the period, and Figure 3.1b shows that for most of the period, physical capital growth was positive. After calculating physical capital stock and its growth, the next step in the estimations would be to determine the order of integration of each of the variables to be used in the different models.

3.3 Unit Root Results

Engle and Granger (1987) show that estimations using non-stationary variables lead to spurious results. Table 3.1 gives the unit root test results at levels of the variables and the critical values at 5 per cent significance level given in parenthesis.

The Augmented Dickey Fuller (ADF) unit root test, the Phillip Perron (PP) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test results on the variables show that GDP growth, physical capital growth and human capital growth are all stationary at levels. Labour growth, GDP per worker and physical capital per worker are all unit root processes and are integrated in order one. To avoid spurious results, the non-stationary variables are made stationary by differencing.

Figure 3.1: Evolution of physical capital stock in Kenya (1982-2006)

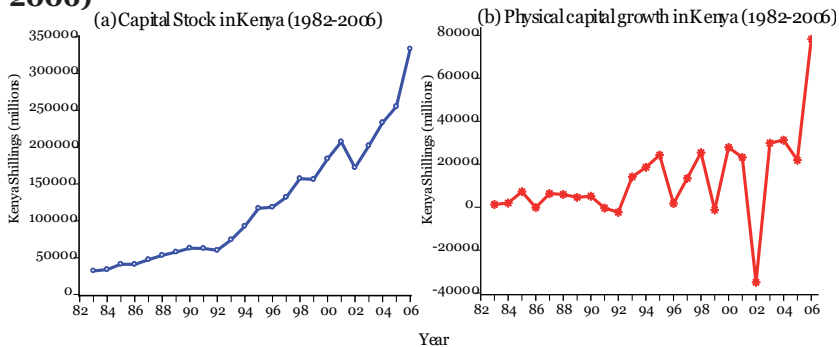


Table 3.1: Unit root results

Variable	ADF	PP	KSPS	Remarks
GDP growth	-2.96(-2.99)	-2.96(-2.99)	0.11(0.46)	I(1)
Physical capital growth	-4.73(-2.99)	-4.71(-2.99)	0.22(0.46)	I(0)
Labour growth	-1.76(-2.99)	-1.76(-2.99)	0.37(0.46)	I(1)
Human capital growth	-3.77(-2.99)	-3.77(-2.99)	0.10(0.46)	I(0)
Capital per worker	-0.075(-3.65)	0.93(-3.61)	0.21(0.14)	I(1)
GDP per worker	-0.29(-3.62)	0.16(-3.61)	0.17(0.14)	I(1)

3.4 Differences in Factor Share Estimates from the Different Approaches

We use four different approaches reviewed in section two to estimate the factor shares and then compare the results for any significant differences in the estimates and model performance. The first approach assumes that the share of physical capital is 0.3 and the share of labour is 0.7 under constant returns to scale. This will be our benchmark model for comparison purposes. In the second, third and fourth approaches, we let the data generating process determine the factor shares and returns to scale. In the second approach, we estimate the Cobb-Douglas technology in its intensive form as given in equation (2) by linearizing and regressing the log of GDP per worker on the log of capital per worker and a time trend. We call this Model 1. A variant of Model 1 is also estimated where the Cobb-Douglas technology is linearized in its extensive form as given in equation (3). The log of GDP is regressed on the log of physical capital, the log of labour and a time trend. We call this Model 2. In the third model, we estimate the factor shares using the growth of GDP, capital growth and labour growth from the Solow model specification given in equation (8). This becomes our Model 3. In Model 3, we do not differentiate between physical and human capital. Model 4 estimates the factor shares from an augmented Solow model specified in equation (11) with physical capital growth, labour growth and human capital growth as the regressors. In this model, capital is either physical or human. The estimation results from the different approaches are given in the next sub-sections.

3.4.1 Factor share estimates from the intensive form technology (Model 1)

In this section, we estimate Model 1 given by equation (2). The results from this estimation are given in Table 3.2.

Table 3.2: Factor shares from the intensive form technology (Model 1)

Dependent variable: Dlog GDP per worker				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
Dlog capital per worker	0.094287	0.077644	1.214345	0.2395
trend	-0.002291	0.002791	-0.820853	0.4219
C	0.132376	0.046238	2.862921	0.0100
AR(1)	0.527619	0.207764	2.539505	0.0200
R-squared	0.405521	Mean dependent var		0.105370
				0.05024
Adjusted R-squared	0.311656	S.D. dependent var		4
				-
				3.36052
S.E. of regression	0.041686	Akaike info criterion		9
				-
Sum squared resid	0.033017	Schwarz criterion		3.163052
		Hannan-Quinn		-
Log likelihood	42.64609 criter.			3.310864
F-statistic	4.320262	Durbin-Watson stat		1.788470
Prob(F-statistic)	0.017536			

This estimation makes one limiting assumption, that production takes place under constant returns to scale but leaves the factor shares to be determined by the data generating process. From the estimation results, equation (2) is then of the form:

$$\ln y_t = 0.132 - 0.0022t + 0.094 \ln k_t + \alpha_t \dots \dots \dots (14)$$

From the results, the share of capital in output is found to be 0.094 and, assuming constant returns to scale, the share of labour in output is therefore $(1 - 0.094) = 0.906$. The results show that the model explains 40 per cent of the variations in the dependent variable. Apart from assuming constant returns to scale, in the next step, another limiting assumption is introduced; that physical capital share is 0.3 as commonly used in various literature (Solow, 1957; Mankiw, Romer and Weil, 1992; Klenov and Rodreguex-Claire, 1997; and Gollin, 1996). The estimation results with these two restrictions are given in Table 3.3.

The results from Table 3.3 show that the proportion of the dependent variable explained by the model when physical capital share is restricted to 0.3 reduces from 40 per cent obtained in Table 3.2 to

Table 3.3: Factor share estimates from the restricted intensive form technology

Dependent variable: Dlog GDP per worker				
	Coefficient	Std. Error	t-Statistic	Prob.
c	0.119798	0.032152	3.726037	0.0013
trend	-0.002802	0.002031	-1.379619	0.1829
AR(1)	0.270435	0.214073	1.263281	0.2210
R-squared	0.206756	Mean dependent var		0.105370
				0.05024
Adjusted R-squared	0.127432	S.D. dependent var		4
				-
S.E. of regression	0.046934	Akaike info criterion		3.159040
				-
				3.010932
Sum squared resid	0.044056	Schwarz criterion		2
		Hannan-Quinn		
Log likelihood	39.32896 criter.			-3.121791
F-statistic	2.606464	Durbin-Watson stat		2.032819
Prob(F-statistic)	0.098643			
Inverted AR Roots	.27			

20 per cent in the restricted model in Table 3.3. Since we are assuming constant returns to scale, the labour share is 0.7. This implies that the restrictions are not appropriate, since they reduce the explanatory power of the model.

3.4.2 Factor share estimates from the extensive form technology (Model 2)

A variant of Model 1 given by equation (3) is estimated here and the results compared to the Model 1 results. The results of the estimation of equation (3) are given in the Table 3.4.

The share of physical capital in production is 0.08 and the share of labour in output is 0.43. Equation (3) therefore, is of the form:

$$\ln y_t = 0.184 - 0.004t + 0.083 \ln k_t - 0.43 \ln l_t + \alpha_t \dots\dots\dots(16)$$

The findings show that capital share remains almost the same as in the intensive form specification results in Table 3.2. There is, however, a substantial change in the labour share from 0.906 implied by the results from Table 3.2, to 0.43. This could be an indication of

Table 3.4: Factor shares from the unrestricted extensive form technology (Model 2)

Dependent Variable: Dlog GDP				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
Dlog Capital	0.083560	0.073900	1.130722	0.2730
Dlog Labour	-0.436548	1.115781	-0.391249	0.7002
Trend	-0.004057	0.003403	-1.192269	0.2486
C	0.184881	0.068778	2.688093	0.0150
AR(1)	0.573885	0.207535	2.765246	0.0128
				0.12344
R-squared	0.498271	Mean dependent var		0
				0.05248
Adjusted R-squared	0.386776	S.D. dependent var		1
				-
				3.35609
S.E. of regression	0.041097	Akaike info criterion		8
				-
Sum squared resid	0.030402	Schwarz criterion		3.109251
		Hannan-Quinn		-
Log likelihood	43.59512	criter.		-3.294016
F-statistic	4.468986	Durbin-Watson stat		1.987677
Prob(F-statistic)	0.011050			

Table 3.5: Returns to scale test in the unrestricted extensive form technology estimates

Wald test			
Test Statistic	Value	df	Probability
F-statistic	1.442582	(1, 18)	0.2453
Chi-square	1.442582	1	0.2297

the exaggeration of the labour share, when constant returns to scale are assumed. The explanatory power of the model is found to be 0.49, which is a significant increase from the 40 per cent and the 20 per cent obtained in the restricted model results in Table 3.2 and 3.3. Testing for constant returns to scale is as shown in Table 3.5.

The (probability values) p-values from the results show that at both 5 per cent and 1 per cent, we cannot reject the null hypothesis of constant returns to scale. Model 2, therefore, suggests that production in Kenya takes place under constant returns to scale with a capital share of 8 per cent, and labour share of 43 per cent in output. When we restrict the capital share to be 0.3 in Model 2, we get the results in the Table 3.6.

Table 3.6: Factor shares from the restricted extensive form technology (Model 2)

Dependent variable: Dlog GDP				
	Coefficient	Std. Error	t-Statistic	Prob.
c	0.152955	0.053047	2.883403	0.0095
Dlog Labour	-0.216675	1.144708	-0.189284	0.8519
trend	-0.003963	0.002538	-1.561741	0.1349
AR(1)	0.278980	0.217804	1.280874	0.2157
R-squared	0.296810	Mean dependent var		0.123440 0 0.052481
Adjusted R-squared	0.185780	S.D. dependent var		1 -
S.E. of regression	0.047356	Akaike info criterion		3.105487 - 2.90801
Sum squared resid	0.042609	Schwarz criterion		0 -
		Hannan-Quinn		3.055822
Log likelihood	39.71310 criter.			2
F-statistic	2.673243	Durbin-Watson stat		2.125509
Prob(F-statistic)	0.076578			

The results from Table 3.6 show an R-squared of 0.29. This is a reduction from the 0.49 obtained from the unrestricted model in Table 3.4. The results imply that the restricted model has a lower explanatory power than the unrestricted model. Testing for constant returns to scale in the restricted model is as shown in Table 3.7.

The results show that at both 5 per cent and 1 per cent, we cannot reject the null hypothesis of constant returns to scale. The restricted extensive form technology estimates give the same results as its unrestricted version, which shows that production in Kenya takes place under constant returns to scale.

3.4.3 Estimating factor shares from the Solow model (Model 3)

In this section, we estimate the factor shares from the Solow Model (Model 3) as specified in equation (8). The results from this estimation are given in Table 3.8. From the results in Table 3.8, equation (8) is of the form:

$$\dot{y}_t = 14.59 + 0.032\dot{k}_t + 0.96\dot{l}_t + \dot{a} \dots\dots\dots(17)$$

Table 3.7: Returns to scale test in the restricted extensive form technology estimates

Wald test:			
Test Statistic	Value	df	Probability
F-statistic	0.641271	(1, 19)	0.4332
Chi-square	0.641271	1	0.4233

The results show that the unrestricted Solow model explains 39 per cent of the variations in GDP growth, with a capital share of 0.032 and labour share of 0.96. The sum of capital and labour shares is $0.032 + 0.960 = 0.992$, which is less than one. To determine whether this sum is statistically different from one (constant returns to scale), we use the Wald's test to test the null hypothesis of constant returns. The test results are reported in the Table 3.9.

The results give a probability value (p-value) of 0.998, implying that we cannot reject the null hypothesis of constant returns to scale. This means that production in Kenya takes place under constant returns to scale. When the Solow model (Model 3) is restricted with a physical capital share of 0.3, the results in Table 3.10 are obtained.

Table 3.8: Estimates of factor shares from the unrestricted Solow model (Model 3)

Dependent variable: GDP growth				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
Capital growth	0.032524	0.035208	0.923767	0.3672
DLabor growth	0.966528	0.388549	2.487532	0.0223
C	14.59497	0.970532	15.03811	0.0000
AR(1)	0.253875	0.230050	1.103561	0.2836
R-squared	0.393409	Mean dependent var		15.29959
				2.03356
Adjusted R-squared	0.297632	S.D. dependent var		9
				4.06093
S.E. of regression	1.704282	Akaike info criterion		6
Sum squared resid	55.18697	Schwarz criterion		4.258413
		Hannan-Quinn		
Log likelihood	-42.70076 criter.			4.110601
F-statistic	4.107532	Durbin-Watson stat		2.095014
Prob(F-statistic)	0.020984			

Table 3.9: Returns to scale test in the unrestricted Solow model (Model 3)

Wald test:			
Test Statistic	Value	df	Probability
F-statistic	6.17E-06	(1, 19)	0.9980
Chi-square	6.17E-06	1	0.9980

Table 10: Factor share estimates from the restricted Solow model (Model 3)

Dependent Variable: GDP growth				
	Coefficient	Std. Error	t-Statistic	Prob.
C	8.211336	0.608751	13.48883	0.0000
Labour growth	0.206731	0.752483	0.274732	0.7863
AR(1)	-0.100745	0.239127	-0.421301	0.6780
R-squared	-1.258596	Mean dependent var		15.29959 2.03356
Adjusted R-squared	-1.484456	S.D. dependent var		9 5.28862
S.E. of regression	3.205344	Akaike info criterion		4
Sum squared resid	205.4846	Schwarz criterion Hannan-Quinn		5.436731
Log likelihood	-57.81917	crit.		5.325872
Durbin-Watson stat	1.963927			

3.4.4 Estimating factor shares from the augmented Solow model (Model 4)

After estimating the Solow model, we augment it to include human capital accumulation as one of the factor inputs as given in equation (11). The results from estimation of the augmented model (11) are given in the Table 3.11.

The results from Table 3.11 imply that equation (11) is of the form:

$$\dot{y}_t = 13.44 + 0.03\dot{k}_t + 0.83\dot{l}_t + 0.05\dot{h}_t + \dot{a}_t \dots\dots\dots(18)$$

The augmented model results show that the share of physical capital in output is 0.037, which is not significantly different from the 0.032 obtained in the Solow model (Table 3.8). The share of labour in output is 0.83, down from 0.96 obtained in the Solow model, while the share of human capital in output is 0.05. The results show that with the augmentation, the explanatory power of the model improves from 39 per cent obtained in the unrestricted version of the Solow model (Table 3.8) to 47 per cent (Table 3.11). These findings are consistent with the findings of Makiew, Romer and Weil (1992), which showed

Table 3.11: Factor shares estimates from the unrestricted augmented model

Dependent Variable: GDP growth				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
Capital growth	0.037693	0.033449	1.126872	0.2746
DLabour growth	0.833774	0.378776	2.201235	0.0410
Human capital growth	0.052519	0.032566	1.612671	0.1242
C	13.44344	1.175102	11.44024	0.0000
AR(1)	0.268192	0.236481	1.134094	0.2716
R-squared	0.470050	Mean dependent var		15.29959
				2.03356
Adjusted R-squared	0.352283	S.D. dependent var		9
				4.012820
S.E. of regression	1.636634	Akaike info criterion		0
Sum squared resid	48.21426	Schwarz criterion		4.259667
		Hannan-Quinn		4.07490
Log likelihood	-41.14743 criter.			2
F-statistic	3.991368	Durbin-Watson stat		7
Prob(F-statistic)	0.017245			

that augmenting the Solow model with human capital accumulation improves the explanatory power of the model. The results, however, find no support for Mankiew, Romer and Weil (1992)'s argument that physical capital has greater impact on income when human capital accumulation is taken into account (Mankiew, Romer and Weil, 1992). The improvement in the explanatory power of the model with the augmentation points to the importance of human capital accumulation in explaining economic growth, other than just the conventional physical capital accumulation and labour growth proposed by the Solow model. The sum of the three factor shares adds up to 0.917, which is less than 1. To test whether this sum is statistically different from one (constant returns to scale), we use the Wald test to test for the null of constant returns to scale, with the results reported in the Table 3.12.

The p-value from the results show that we cannot reject the null hypothesis of constant returns to scale. This implies that the sum 0.917 is not statistically different from one. Therefore, production in Kenya, considering the three factors of production (physical capital, human capital and labour) as inputs, takes place under constant returns to scale. In the next estimation, we restrict physical capital share in the

Table 12: Constant returns to scale test in the augmented Solow Model (Model 3)

Wald test			
Test Statistic	Value	df	Probability
F-statistic	0.042811	(1, 18)	0.8384
Chi-square	0.042811	1	0.8361

augmented model to be 0.3. The results of this restricted estimation are given in the Table 3.13.

The main observations from the preceding analysis are: First, in all cases of the unrestricted estimations, the share of physical capital is found to be less than 10 per cent, with the highest share obtained in the intensive Model 2 at 9 per cent. An estimation which assumes 30 per cent as the share of physical capital would therefore not be supported by the data generating process, leading to biased results. Secondly, in all cases, the explanatory power of the model decreases when restrictions on factor shares are imposed, implying that the restrictions leads to misspecification of the growth model and are therefore not appropriate. Third, in all cases, it is possible to test for the null hypothesis of constant returns to scale empirically; therefore, prior assumptions on the same are unnecessary.

Does it matter for growth accounting estimates whether the values of factor shares are obtained from the data generating process or whether they are assumed apriori? The next section examines the consequences of using prior factor share restrictions on growth accounting estimates. The consequences of using the different factor share estimates to calculate TFP growth are first examined, then the consequences of using them to account for the sources of economic growth.

3.5 Differences in TFP Growth Estimates Resulting from Using Different Factor Share Estimates

Total factor productivity (TFP) can easily be estimated from regression (8) as the fitted values from the regression residuals. This approach (as we saw earlier) does not require that we know the values of the factor shares before estimating. The factor shares are the estimated regression parameters from the regression of (8). The approach is also very informative, since we can determine whether production takes place under decreasing, constant or increasing returns to scale just by looking

Table 3.13 Factor shares from the restricted augmented model

Dependent variable: GDP growth				
	Coefficient	Std. Error	t-Statistic	Prob.
C	6.406549	1.284039	4.989374	0.0001
Labour growth	-0.122044	0.765133	-0.159506	0.8750
Human capital growth	0.092339	0.058839	1.569347	0.1331
AR(1)	-0.168157	0.246611	-0.681871	0.5035
R-squared	-1.008061	Mean dependent var		15.2995
Adjusted R-squared	-1.325123	S.D. dependent var		2.03356 9 5.25800
S.E. of regression	3.100859	Akaike info criterion		6
Sum squared resid	182.6912	Schwarz criterion Hannan-Quinn		5.455483
Log likelihood	-56.46707 criter.			5.307671
Durbin-Watson stat	2.031876			
Inverted AR Roots	-0.17			

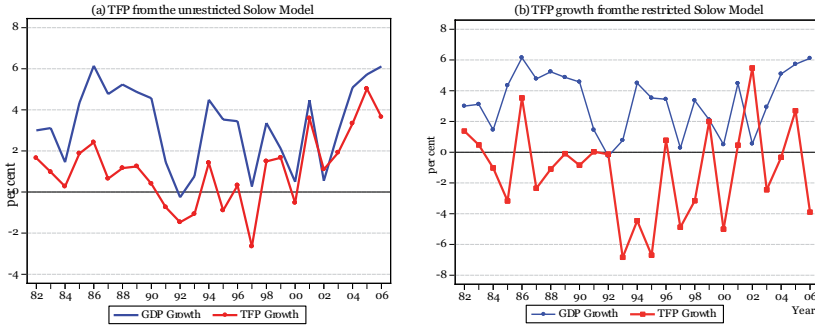
at the sum of the coefficients of capital growth and labour growth obtained from the regression of (8) and testing whether the sum is statistically different from one. The other approach of calculating TFP, assuming we want to restrict physical capital share to be 0.3 and labour share to be 0.7 (assuming constant returns to scale), is to transform equation (8) to:

$$\dot{a}_t = \dot{y}_t - 0.3\dot{k}_t - 0.7\dot{l}_t \dots\dots\dots(19)$$

With data on GDP growth \dot{y}_t , physical capital growth \dot{k}_t and labour growth \dot{l}_t ; TFP growth \dot{a}_t can be calculated simply by subtracting the factor growths multiplied by their respective shares in output from output growth. Figure 2 gives the graphs of TFP growth calculated from the unrestricted Solow model and one from the restricted Solow model.

Figure 3.2a shows the close relationship between TFP growth obtained without any restrictions on the factor share and GDP growth. It is clear from the graph that whenever TFP growth fell, GDP growth rate fell as well, and whenever TFP growth rose, GDP growth rate rose as well. Figure 3.2b on the other hand, plots the trend in TFP growth calculated assuming that physical capital share is 0.3 and compares it with the trend in GDP growth in Kenya over the same period. As can be seen from Figure 3.2, TFP growth, when no factor share restrictions are imposed, traces GDP growth much better than it does in Figure

Figure 3.2: TFP growth at different capital shares vs GDP growth in Kenya



3.2b with the restrictions. This could imply that TFP growth obtained from the unrestricted model is a much better explanatory variable of GDP growth than the one calculated from the restricted model with a physical capital share of 0.3. It could therefore be expected that using the unrestricted model will result in a higher explanatory power of the model than using the restricted model. The results in Figure 3.2 therefore, imply that the choice of the factor share restrictions is important for TFP growth results. Choosing a wrong factor share is thus likely to lead to wrong TFP growth estimates.

In the next section, we use the different estimates of TFP growth (obtained using the different factor shares calculated from the different approaches in section 3.4) to account for the sources of economic growth in Kenya and see how the results change as different TFP growth figures are used.

3.6 Structural VAR (SVAR) Estimates and Sources of Growth

This section determines the differences in growth accounting estimates that result from using the factor shares obtained from the different approaches. To do this, we use Structural Vector Autoregressions (SVAR) and variance decomposition generated from the estimated SVAR to account for the percentage change in GDP growth that are attributable to the different factor growths.

3.6.1 SVAR model specification and identification

The single equation regression from which the SVAR is developed is

given by equation (8) as:

$$\dot{y}_t = \alpha \dot{k}_t + \lambda \dot{l}_t + \dot{a}_t$$

Stationarity results on the variables in equation (8) are given in Table 3.1. The results show that physical capital growth is I(0), GDP growth is I(0), labour growth is I(1) while human capital growth is I(0). TFP growth estimated without any restrictions on the factor shares of (2.8) is I(0), TFP growth estimated using capital share of 0.3 and labour share of 0.7 (calculated from equation 19) is I(1), while TFP growth calculated from the unrestricted augmented Solow model (11) is I(0).

It is important that the variables that are not stationary be made stationary before any estimation is commenced to avoid spurious results. This is done by differencing the I(1) variables. Enders (2005) notes that if the variables in a model are cointegrated, then specifying and estimating the model, excluding the long run relationship contained in the error correction term leads to misspecification. In our case however, since physical capital growth is an I(0) variable, we do not expect that it will be cointegrated with labour growth, which is I(1). Therefore, any model with the two variables together, cannot have an error correction representation. In this case, we do not expect to have a misspecification error by omitting the error correction term from such a model. We can have a slight modification of the Solow model given in equation (8) to allow for other unobserved random factors to explain economic growth so that:

$$\dot{y}_t = \alpha \dot{k}_t + \lambda \Delta \dot{l}_t + \dot{a}_t + \varepsilon_t, \dots \dots \dots 20)$$

where Δ is the first difference operator and ε_t is the random component of GDP growth that is not accounted for by the growth in factors, and TFP growth. In this case, \tilde{a}_t is not the regression residual. Equation (20) is then used to specify the unrestricted vector autoregression (VAR) model.

As is well known, the unrestricted VAR is not identified and cannot therefore be used for any meaningful policy inference. Structural identification of the VAR requires the imposition of theoretical restrictions necessary to identify and put economic meaning to the unrestricted VAR. To identify the unrestricted VAR, the main question we ask is whether we have enough apriori information concerning the variables? The unrestricted VAR representation of the relationships in equation (20) assuming one lag, is given as:

$$\begin{bmatrix} 1 & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & 1 & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & 1 & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix} \begin{bmatrix} \dot{y}_t \\ \dot{k}_t \\ \Delta \dot{l}_t \\ \dot{a}_t \end{bmatrix} + \begin{bmatrix} -\beta_{11} & -\beta_{12} & -\beta_{13} & -\beta_{14} \\ -\beta_{21} & -\beta_{22} & -\beta_{23} & -\beta_{24} \\ -\beta_{31} & -\beta_{32} & -\beta_{33} & -\beta_{34} \\ -\beta_{41} & -\beta_{42} & -\beta_{43} & -\beta_{44} \end{bmatrix} \begin{bmatrix} \dot{y}_{t-1} \\ \dot{k}_{t-1} \\ \Delta \dot{l}_{t-1} \\ \dot{a}_{t-1} \end{bmatrix} = \begin{bmatrix} \varepsilon_{\dot{y}_t} \\ \varepsilon_{\dot{k}_t} \\ \varepsilon_{\Delta \dot{l}_t} \\ \varepsilon_{\dot{a}_t} \end{bmatrix} \quad (21)$$

where $x_t = [\dot{y}_t, \dot{k}_t, \Delta \dot{l}_t, \dot{a}_t]$

In compact form, the model above is given as:

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$$

In the reduced form;

$$x_t = B^{-1} \Gamma_0 + B^{-1} \Gamma_1 x_{t-1} + B^{-1} \varepsilon_t$$

$$\Rightarrow x_t = A_0 + A_1 x_{t-1} + e_t$$

where $A_0 = B^{-1} \Gamma_0$, $A_1 = B^{-1} \Gamma_1$ and

$$e_t = B^{-1} \varepsilon_t \dots \dots \dots (22)$$

In the subsequent discussion, we will refer to the matrix with α_s as the B matrix. The problem now is to take the observed values of e_t in (22) and restrict the B matrix so as to recover the unobserved ε_t from the residuals e_t . Following the AB model identification approach proposed by Gianni and Giannini (1997), we have:

$$Ae_t = B\varepsilon_t \dots \dots \dots (23)$$

where B is a $n \times n$ diagonal matrix and ε_t represents a matrix of structural shocks. The structural innovations ε_t are assumed to be orthonormal, that is its covariance matrix is an identity matrix. The assumption of orthonormal structural innovations imposes the following identifying restrictions on A and B:

$$A \Sigma_e A' = B B' \dots \dots \dots (24)$$

The symmetry of both sides of (24) imposes $n(n+n)/2$ restrictions on the $2n^2$ unknown elements of both A and B. This comes to $(2n^2 - n(n+n)/2) = n(3n-1)/2$ additional restrictions required for exact identification of (21) (Amisano and Gianinni, 1997).

3.6.2 Theoretical restrictions

To identify the structure of the model, it is necessary to go back to economic growth theory to identify the endogenous and exogenous variables in the model, or have some expectations on the signs and the magnitudes of some coefficients. The first approach that forms our baseline model assumes that capital share in output is 30 per cent, such as is used in other studies (Mankiw, Romer and Weil, 1992; Klenov and Rodreguez-Claire, 1997; and Gollin, 1996) so that α_{12} in equation (21) equals to 0.3. This means that the labour share in output, assuming constant returns to scale, is 0.7 i.e. $\alpha_{13}=0.7$. In the second approach, we leave the factor shares to be determined by the data generating process and see how the results of growth accounting are different when the two approaches are used separately. In the unrestricted model, the coefficients representing the factor shares will be left free to be determined by the data generating process.

To determine the endogenous and exogenous variables in the model, we adopt Mankiw, Romer and Weil (1992) formulation and assume that labour and technology grow exogenously at the constant rates n and g , respectively that is $L_t = L(0)e^{nt}$ and $A_t = A(0)e^{gt}$. This means that all the elements in the last and the second last rows of the B matrix in (equation 21) are zeros, except for the principle diagonal which are ones. From the Solow model, the evolution of capital is governed by $\dot{k}_t = sf(k_t) - (n + g + \delta)k_t$ but $f(k_t) = A_t k_t^\alpha$ so that

$$\dot{k}_t = sA_t k_t^\alpha - (n + g + \delta)k_t \dots\dots\dots(25)$$

where δ is the depreciation rate and s is a constant fraction of output that is invested, or the savings propensity. Equation 25 implies that k converges to a steady-state value k^* given as:

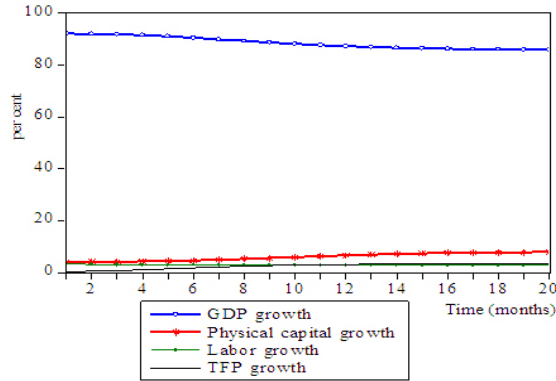
$$\Rightarrow k^{*1-\alpha} = \frac{sA_t}{(n + g + \delta)} = \left(\frac{sA_t}{(n + g + \delta)} \right)^{\frac{1}{1-\alpha}} \dots\dots\dots(26)$$

Taking logs we have:

$$\ln(k^*) = \frac{1}{1-\alpha} \ln(s) + \frac{1}{1-\alpha} \ln(A_t) - \frac{1}{1-\alpha} \ln(n + g + \delta) \dots\dots\dots(27)$$

The relationship in equation (27) shows that physical capital depends on the savings propensity, growth of labour, technical progress and the rate of depreciation. We do not have savings propensity and the rate

Figure 3.3: Sources of GDP growth from the restricted Solow model



of depreciation as variables in our model, therefore physical capital is determined by labour growth and technical progress.

With physical capital share at 0.3 and labour share of 0.7, the B matrix is of the form:

$$\begin{bmatrix} 1 & 0.3 & 0.7 & -\alpha_{14} \\ 0 & 1 & -\alpha_{23} & -\alpha_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots\dots\dots(28)$$

In the unrestricted model where no coefficient restrictions on capital share and labour share are imposed, the B matrix is given as:

$$\begin{bmatrix} 1 & -\alpha_{12} & -\alpha_{13} & -\alpha_{14} \\ 0 & 1 & -\alpha_{23} & -\alpha_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots\dots\dots(29)$$

The estimation procedure of model (21) with the restrictions (27) and (28) follows by first estimating the unrestricted VAR (21) then an appropriate lag length is chosen using the different information criterion. The unrestricted VAR is then re-estimated using the optimal lag length chosen and, thereafter, the theoretical restrictions imposed. From the SVAR results, variance decompositions of the changes in GDP growth are generated.

3.7 Differences in the Growth Accounting Results

3.7.1 Sources of GDP growth from the restricted Solow model

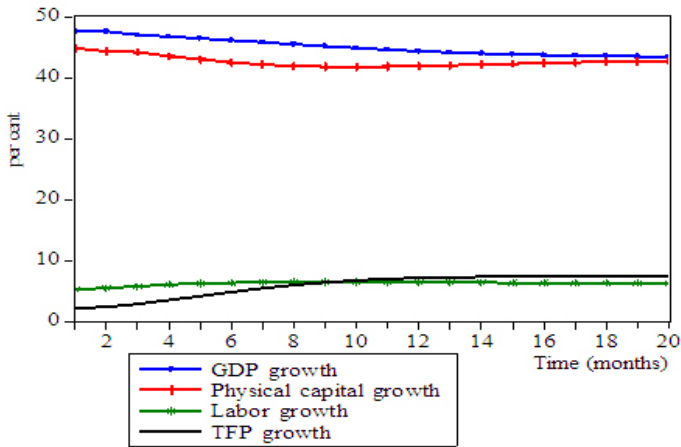
If we restricted capital share to 0.3 in the Solow model with the B matrix given by matrix (28), the variance decomposition of GDP growth obtained from the restricted estimation is given in Figure 3.3.

Figure 3.3 shows that if we assumed that the share of physical capital in output is 0.3, then other than own shocks, physical capital growth accounts for the largest share of the changes in the GDP growth in Kenya (between 4% and 7% over the time horizon of twenty months –Appendix Table A1). The results further show that in the first ten months, labour growth was the second highest source of GDP growth in Kenya at between 3 per cent and 2 per cent. The contribution of labour growth, however, is overtaken after the tenth month by the contribution of TFP growth as the second highest contributor to GDP growth. It is important to note that none of the factors in this case account for more than 7 per cent of the total GDP growth changes, with GDP's own shocks contributing the bulk of the changes in GDP growth at between 92 per cent in the first month, and 86 per cent in the last month. It is also clear from the results that as the months go on, the contribution of own shocks decrease as factor contributions increase, particularly from physical capital growth and TFP growth. If it was true that capital share in Kenya is 0.3 and labour share is 0.7, and that production is undertaken under constant returns to scale, then we can argue that physical capital is the major source of economic growth in Kenya. However, we have no basis to argue that the share of physical capital in output in Kenya is 0.3. If in fact it is not, then the variance decomposition would obviously be misleading for policy purposes. It would even be more misleading when one is carrying out cross-country analysis in a panel situation, for instance to argue that different countries would have the same capital share of say 0.3. How then would the results change if capital share is assumed, unknown and left to the data generating process to determine?

3.7.2 Sources of GDP growth from the unrestricted Solow model

The variance decomposition of the changes in GDP growth generated from the estimation of the unrestricted Solow model with the B matrix

Figure 3.4: Sources of GDP growth in Kenya from the unrestricted Solow model



(29) is given in Figure 3.4. Appendix Table A2 gives the variance decomposition table from this estimation.

Figure 3.4 shows that other than own shocks, physical capital growth accounts for the highest percentage of the changes in income growth. This is the same result that we had when we had the restriction of physical capital at 0.3. However, physical capital growth now accounts for between 44 per cent in the first month and 42 per cent of the GDP growth in the 20th month. This is a big increase from the previous 4 per cent and -7 per cent obtained when the share is restricted to 0.3 (Figure 3.3). The results further show that the second most important source of economic growth is labour growth up to the tenth month and after the tenth month, it is overtaken by TFP growth as the second most important source of GDP growth. The results are again consistent with the results obtained from the restricted model in Figure 3.3, which showed that labour growth dominates TFP growth up to the tenth month, and vice versa. The only interesting change when the share is not restricted is that the contribution of labour growth increases from between 3 per cent and -2 per cent to between 5 per cent and -6 per cent over the period. The contribution of TFP growth increases from between 0.4 per cent and -3 per cent to between 2 per cent and -7 per cent. This clearly shows that the restrictions on factor shares grossly underestimate the contribution of the factors to economic growth. On the other hand, the same restrictions overestimate the contribution of own shocks. As can be seen from the results in Figure 3.3 and 3.4, the contribution of GDP own shocks reduced in the unrestricted model to

between 47 per cent and 43 per cent down from 92 per cent and 86 per cent obtained in the restricted model over the sample period.

3.7.3 Sources of GDP growth from the unrestricted augmented Solow model

As seen earlier, allowing for human capital accumulation will change the technology to be of the form:

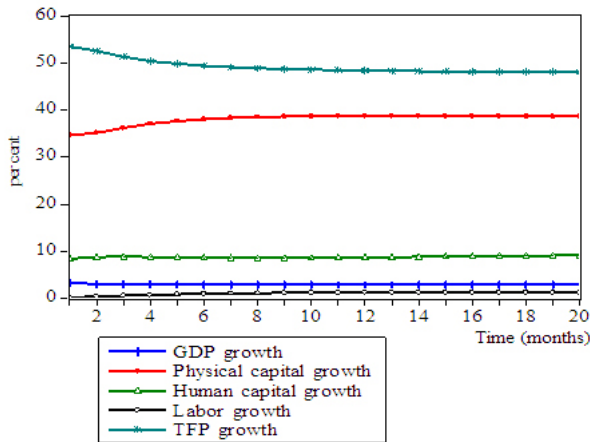
$$Y_t = A_t e^{\beta t} K_t^\alpha H_t^\delta L_t^{1-\alpha-\delta} \quad (30)$$

The estimatable form of equation (30) is given by equation (11). With data on TFP growth calculated as the residuals from (11), we can then account for GDP growth in the unrestricted augmented Solow model. We assume that human capital is determined by income and labour growth. The more income a country has, the better it is able to provide training facilities, including equipped universities for training. It is also expected that increased labour growth in the form of increased employment opportunities and incentives at the workplace will act as an enticement for the population to seek further training so as to take up the positions. Increased unemployment rates, on the other hand, discourage people from seeking further training. As noted earlier, human capital accumulation is measured as the total enrolment in higher learning institutions. It is proxied only with higher education enrolment other than primary and high school enrolment, since enrolment in higher education institutions creates a pool of skilled labour, which is instrumental in contributing to economic growth activities. It is, for instance, difficult to get job placements with primary school or high school certificates and, therefore, increases in primary and secondary school enrolment and completion rates have no added marginal increase on economic growth. Assuming no time trend βt , the B matrix of the unrestricted augmented model is therefore given as:

$$\begin{bmatrix} 1 & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} \\ \alpha_{21} & 1 & 0 & \alpha_{24} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \alpha_{51} & 0 & \alpha_{53} & 0 & 1 \end{bmatrix} \quad (31)$$

The variance decomposition of GDP growth from the SVAR with the B matrix (31) is given in the Figure 3.5. The variance decomposition

Figure 3.5: Sources of GDP growth from the unrestricted augmented Solow model



tables from this estimation are given in Appendix Table A3.

From the variance decomposition results in Figure 3.5, there are some very surprising differences from the Solow model analyzed previously. The results show that with human capital accumulation, TFP growth is the major source of economic growth in Kenya. This is different from the case without human capital accumulation (both with and without restrictions), which showed that physical capital is the major source of economic growth in Kenya. TFP growth now accounts for between 53 per cent in the first month to 48 per cent in the 20th month of economic growth, as can be seen in the Figure 3.5 and Appendix Table A3. In the previous model without human capital accumulation, TFP was found to account for between 2 per cent and -7 per cent. This shows that the model without human capital accumulation underestimates the role of TFP in economic growth. The results further show that the second highest contribution comes from physical capital growth, with a contribution of between 34 per cent and 38 per cent over the 20-month forecast horizon. This is a reduction in the contribution of physical capital when human capital accumulation is included in the model. The contribution of physical capital reduces from between 44-42 per cent that was obtained for the Solow model without human capital accumulation. This result contradicts the result obtained by Mankiew, Romer and Weil (1992) that physical capital contribution to GDP growth increases with the augmentation. The decrease in the contribution of physical capital could be attributed to the fact that without the augmentation, physical capital is over-estimated, since it includes all

elements of capital as well as human capital. With the augmentation, therefore, human capital component is separated from physical capital, hence the reduction in the contribution of physical capital growth to output growth. Excluding human capital accumulation in the model, therefore, seems to overestimate the contribution of physical capital to economic growth.

The results show that human capital growth accounts for between 8 per cent of economic growth in the first month to 9 per cent in the 20th month. Own shocks account for between 3 per cent and 2.9 per cent down from between 47 per cent and 43 per cent in the Solow model without human capital accumulation. This again shows that without including human capital in the model, the role of own shocks are grossly exaggerated. Labour growth accounts the least to GDP growth, with a contribution of between 0.2 per cent and 1.2 per cent over the period. This was also over-estimated by the Solow model without human capital accumulation.

4. Summary and Conclusions

This paper determines whether factor share restrictions are appropriate in growth accounting estimations. Several studies have used some prior specific assumptions on the magnitude of factor shares and specific assumptions on the scale of production. Solow (1956), for instance, used 0.3 as the share of physical capital in output and assumed that production takes place under constant returns to scale. Several studies have quoted Solow and used the same restrictions without taking care whether or not those restrictions make sense in their own studies and whether the data generating process of those countries under study support the assumptions. More serious assumptions have been made in cross-country studies, that factor shares are the same across countries. This obviously is wrong and would most probably lead to misleading growth accounting estimates. This can be disastrous for policy advice.

We estimate both the restricted and the unrestricted versions of the Solow model and examine whether the restrictions have any significant consequences for the resulting growth accounting estimates. We also augment the Solow model with human capital accumulation and estimate both the restricted and the unrestricted versions of the augmented model to examine whether the augmented model is better in terms of its explanatory power than the classical Solow model.

The results show that in all the unrestricted estimations, the share of physical capital is less than 10 per cent, implying that an estimation which imposes 30 per cent as the share of physical capital would not be supported by the data generating process, leading to biased results. The results further show that the explanatory power of the model decreases when restrictions on factor shares are imposed, implying that the restrictions lead to misspecification of the growth model and are therefore not appropriate. The explanatory power of the model is found to increase from 39 per cent in the unrestricted Solow model to 47 per cent in the augmented model, implying that the augmented model should be more preferred when explaining the sources of economic growth.

The variance decomposition from the structural VAR model, shows that the contributions of all the factor growths are greatly improved in the unrestricted Solow model. Therefore, imposing the restrictions on factor shares underestimates the contribution of the factors to economic growth. On the other hand, the same restrictions overestimate the

contribution of own shocks. The findings also show a reduction in the contribution of physical capital growth and labour growth when the Solow model is augmented with human capital accumulation. Excluding human capital accumulation in the growth model seems to overestimate the contribution of physical capital growth and labour growth to economic growth. These findings do not agree with the findings of Mankiew, Romer and Weil (1992), who argue that physical capital has greater impact on income when human capital accumulation is taken into account (Mankiew, Romer and Weil, 1992). The results also show that without including human capital in the model, the role of GDP's own shocks as a source of economic growth is exaggerated. The findings therefore suggest that growth models that are intended to be used for policy purposes need to incorporate human capital as one of the factor inputs.

References

- Gollin, D. (1996), "Getting income shares right: Accounting for the self-employed", Williamstown, MA, the MIT Press.
- Klenow Peter J. and Andrés Rodríguez-Clare (1997) "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" NBER Macroeconomics Annual, 12: 13-103.
- Mankiew, N. G., D. Romer and D. N. Weil (1992), "A contribution to the Empirics of Economic growth", Quarterly Journal of Economics 107 (2): 407-437.
- Solow, R. M. (1956). 'A contribution to the theory of economic growth', Quarterly Journal of Economics 70: 65-94.
- World Bank (2000), "Measuring growth in total factor productivity", Poverty Reduction and Economic Management (PREM) Notes, No. 42.

Appendix

Table A1: Variance decomposition of the Solow model when capital share is 0.3

Period	S.E.	Shock1	Shock2	Shock3	Shock4
1	0.030368	92.14978	4.154469	3.272626	0.423129
2	0.030995	91.98806	4.295572	3.172487	0.543886
3	0.031379	91.91030	4.218301	3.095575	0.775827
4	0.031901	91.55816	4.341486	2.998507	1.101847
5	0.032084	91.10464	4.467503	2.972908	1.454946
6	0.032259	90.55188	4.672903	2.949897	1.825325
7	0.032421	89.93774	4.958009	2.928552	2.175697
8	0.032561	89.32457	5.282203	2.910479	2.482750
9	0.032691	88.73612	5.635146	2.892214	2.736524
10	0.032805	88.19711	5.993875	2.874723	2.934291
11	0.032904	87.72216	6.338977	2.858764	3.080097
12	0.032988	87.31533	6.658333	2.844724	3.181609
13	0.033057	86.97575	6.943540	2.832899	3.247813
14	0.033113	86.69841	7.190663	2.823328	3.287596
15	0.033158	86.47610	7.399173	2.815884	3.308839
16	0.033192	86.30083	7.570823	2.810338	3.318006
17	0.033219	86.16464	7.708891	2.806406	3.320063
18	0.033240	86.06015	7.817475	2.803791	3.318586
19	0.033255	85.98085	7.900968	2.802210	3.315970
20	0.033267	85.92122	7.963697	2.801409	3.313675
Factorization: Structural					

Table A2 : Variance decomposition of the unrestricted Solow model

Period	S.E.	Shock1	Shock2	Shock3	Shock4
1	0.023299	47.73692	44.94093	5.222026	2.100120
2	0.023765	47.71228	44.42595	5.481174	2.380592
3	0.024203	47.10661	44.29940	5.760854	2.833133
4	0.024629	46.83872	43.61194	6.058834	3.490506
5	0.024787	46.54500	43.09851	6.220612	4.135869
6	0.024928	46.24066	42.61197	6.344034	4.803331
7	0.025055	45.92108	42.23486	6.418697	5.425367
8	0.025170	45.58456	42.00522	6.451574	5.958652
9	0.025276	45.25944	41.88550	6.460405	6.394659
10	0.025373	44.95612	41.86166	6.452236	6.729980
11	0.025458	44.68344	41.90889	6.434300	6.973375
12	0.025532	44.44626	42.00200	6.412012	7.139724
13	0.025594	44.24442	42.12170	6.388692	7.245186
14	0.025645	44.07576	42.25214	6.366557	7.305540
15	0.025687	43.93681	42.38172	6.346870	7.334605
16	0.025721	43.82349	42.50268	6.330222	7.343603
17	0.025749	43.73181	42.61032	6.316761	7.341109
18	0.025770	43.65805	42.70235	6.306348	7.333259
19	0.025788	43.59893	42.77826	6.298674	7.324135
20	0.025802	43.55169	42.83877	6.293347	7.316199

Table A3 - Variance decomposition of the augmented Solow model

Period	S.E.	Shock1	Shock2	Shock3	Shock4	Shock5
1	0.022362	3.247543	34.64284	8.431629	0.205551	53.47243
2	0.022860	3.108154	35.15922	8.803673	0.398842	52.53011
3	0.023331	3.059446	36.17500	8.869497	0.607205	51.28885
4	0.023749	3.002099	37.08345	8.801079	0.737465	50.37591
5	0.023905	2.970215	37.61780	8.715122	0.884971	49.81189
6	0.024017	2.946899	38.02880	8.634674	0.997872	49.39176
7	0.024096	2.928713	38.31906	8.584591	1.082420	49.08521
8	0.024151	2.915380	8.50008	8.572148	1.148240	48.86415
9	0.024193	2.905625	38.61658	8.587371	1.194652	48.69578
10	0.024225	2.899033	38.68524	8.623530	1.226227	48.56597
11	0.024251	2.895040	38.72049	8.673678	1.246869	48.46392
12	0.024271	2.893039	38.73390	8.731486	1.259306	48.38227
13	0.024288	2.892521	38.73280	8.792567	1.266112	48.31600
14	0.024302	2.893022	38.72251	8.853761	1.269243	48.26146
15	0.024314	2.894154	38.70676	8.912924	1.270145	48.21602
16	0.024323	2.895618	38.68809	8.968737	1.269864	48.17769
17	0.024332	2.897191	38.66824	9.020452	1.269106	48.14501
18	0.024340	2.898719	38.64838	9.067724	1.268322	48.11685
19	0.024346	2.900105	38.62926	9.110482	1.267768	48.09238
20	0.024352	2.901295	38.61134	9.148839	1.267569	48.07096
Factorization: Structural						